Vol. 45, No. 1 DUKE MATHEMATICAL JOURNAL© March 1978

SOME BIRATIONAL INVARIANTS FOR ALGEBRAIC REAL HYPERSURFACES

S. M. WEBSTER

Introduction.

In this paper we consider real hypersurfaces M defined by real polynomial equations in complex projective space $\mathbb{P}_n$. Our aim is to attach to such M invariants under the pseudo-group of birational mappings of $\mathbb{P}_n$. This is of interest because any biholomorphic mapping between algebraic real hypersurfaces with non-degenerate Levi forms must be algebraic [3]. Also, conditions were given in [3] which ensure that the mapping must be birational.

The first section of this paper is concerned with rational anti-polarities. This is a generalization of the concept of the anti-polarity induced by a non-degenerate hermitian form. Associated to a real hypersurface M as above there is a rational anti-polarity which is locally invariant under biholomorphic transformation. This idea goes back essentially to B. Segre [2]. The function sending a point to its polar variety gives an immersion of a non-degenerate M into a real hyperquadric.

In the second section we consider birational mappings between irreducible anti-polarities. The results are applied in the third section to a family of polynomial deformations of the unit sphere in $\mathbb{C}^n$. We show that biholomorphic equivalence reduces to affine equivalence for this family. As a corollary we get a classification theorem for certain central quadrics in $\mathbb{C}^n$. This provides a generalization of the ellipsoid classification theorem of [3].

1. Rational anti-polarities.

Consider complex projective space $\mathbb{P}_n$ with homogeneous coordinates $\zeta = (\zeta_0, \cdots, \zeta_n)$. A non-zero real polynomial $R = R(\zeta, \bar{\zeta})$ will be called *bi-homogeneous of degree m* if $R(s\zeta, t\zeta) = (s\bar{t})^m R(\zeta, \bar{\zeta})$ for all complex s, t. We let $\bar{\eta} = (\bar{\eta}_0, \cdots, \bar{\eta}_n)$ and consider $R(\zeta, \bar{\eta})$. The *polar variety* of the point η relative to R is

$$Q_\eta = \{\zeta : R(\zeta, \bar{\eta}) = 0\},$$

which is a complex hypersurface of degree m in $\mathbb{P}_n$. The Q_η make up a non-linear system of varieties in general.

We shall call the correspondence $\eta \to Q_\eta$ the *rational anti-polarity* associated to R or the *R-anti-polarity*. Since $R(\zeta, \bar{\zeta})$ is real, it follows that $R(\zeta, \bar{\eta}) = 0$ if

Received August 12, 1977. Partially supported by NSF.