THE KRULL-AKIZUKI THEOREM IN THE ABSTRACT

EDUARDO R. BASTIDA

1. Introduction. Let D be a Noetherian domain of dimension one and let J be an overring of D. In [6] W. Krull proved that if J is the integral closure of D, then J is Noetherian of dimension one. In [1] Y. Akizuki generalized this result to overrings J of D that are integral over D by showing that for each nonzero ideal A of J, J/A is a finitely generated $D/(D \cap A)$-module. M. Nagata [7; p. 115] refers to the following more general result as the Krull-Akizuki theorem. *Let D be a Noetherian domain of dimension one and with K as its quotient field. If L is a finite algebraic extension of K and if J is a subring of L containing D, then for each nonzero ideal A of J, J/A is a finitely generated $D/(D \cap A)$-module.* This paper is devoted to the development of a general theory relating the structure of a pair of rings R and S, with R a subring of S, such that the conditions of the Krull-Akizuki theorem are satisfied, that is, S/A is a finitely generated $R/(R \cap A)$-module for each nonzero ideal A of S. If the preceding condition is satisfied, we say that *S is a* KA-*extension of R or that R has property* KA *with respect to S.*

Throughout this work the word ring means commutative ring with identity, and integral domain means a ring with no nonzero zero divisors. If R is a subring of S, then we assume that R contains the identity element of S. Moreover, we say that S is an *overring* of R if S is a subring of the total quotient ring of R containing R.

We consider in Section 2 the KA-property in the general case and establish some of the elementary results about property KA. Section 3 is devoted to the study of the KA-property in the case where the rings under consideration are integral domains. This case is of primary interest, for if S is a KA-extension of a ring R and if S is not an integral domain, then S is a finitely generated R-module. This, however, need not be true if we assume that S is an integral domain.

2. The KA-**property.** Our first objective is to show that if S is a KA-extension of R and if S is not an integral domain, then S is a finitely generated R-module. We begin by listing some results that will be useful throughout this paper. In most cases the proofs are elementary and will be omitted.

Result 1. Let R be a subring of S and A an ideal of S. Then S/A is a finitely generated $R/(R \cap A)$-module if and only if S/A is a finitely generated R-module.

Received November 10, 1972. This paper represents a portion of the dissertation of the author, written under the direction of Professor Robert Gilmer of Florida State University.