ON MAPS WHICH PRESERVE ALMOST PERIODIC FUNCTIONS

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In [1] Eymard poses the question, "What maps ρ from a topological group G to a topological group H have the property that $f \circ \rho$ is almost periodic on G whenever f is almost periodic on H?" Under the assumptions that G is locally compact and H is locally compact abelian (i) such maps are characterized by Lemma 2.1 and Remark 2.2, (ii) those maps with the closure of $\rho(G)$ compact are characterized as $AP(G, H)$ (Definition 1.2) in Theorem 3.6 and (iii) Theorem 2.3 states that such maps must be continuous. If one assumes further that G is abelian and connected, then ρ is the sum of a continuous homomorphism and a function in $AP(G, H)$ (Theorem 4.4). In this case, a complete description of $AP(G, H)$ is given in the statement of the theorem and Remark 3.8.

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1. Preliminaries and notations. All topological spaces are assumed to satisfy the Hausdorff separation axiom. $C(X)$ will denote the set of bounded continuous complex-valued functions endowed with the sup norm and if X is a topological group, f_y will denote the function $f_y(x) = f(y + x)$ for all $x \in X$. For G a topological group, $AP(G)$ is the set of almost periodic functions on G, i.e., $f \in C(G)$ such that $\{f_g\}_{g \in G}$ is totally bounded. There is associated with G a compact topological group bG, the Bohr compactification of G, and a continuous homomorphism α taking G onto a dense subgroup of bG such that $f \in AP(G)$ iff there is an $\bar{f} \in C(bG)$ such that $f(g) = \bar{f}(\alpha(g))$ for all $g \in G$. If G is locally compact abelian (denoted LCA) with character group Γ, then bG is obtained by endowing Γ with the discrete topology and taking its character group, α is one to one and $AP(G)$ is the uniform closure of the complex linear span of the continuous characters of G. See [4] and [5].

Definition 1.1. Let G and H be topological groups. A map $\rho : G \to H$ is said to preserve almost periodic functions iff $AP(H) \circ \rho \subset AP(G)$. Denoted ρ is a.p.p.

Definition 1.2. Let G and H be topological groups. A continuous map $f : G \to H$ is said to be a group valued almost periodic function iff for each neighborhood V of the identity of H there exists a finite set $\{g_1, \cdots, g_n\} \subset G$ such that for each $g \in G$ there is an i, $1 \leq i \leq n$, such that $f(g + x) - f(g_i +$

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