AN APPLICATION OF AN ESTIMATE OF THE RIEMANN MAPPING FUNCTION FOR CERTAIN STAR-SHAPED DOMAINS

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Let u denote a function which is harmonic in a simply-connected domain G and suppose that u is bounded in absolute value throughout G by a known positive constant M. Let D denote a simply connected domain contained in G and suppose that u is bounded in absolute value throughout D by a known positive constant m, where m is less than M. It is well known [3], [4], [5], [7], [8] that for any compact set S in G, there exist constants $K = K(M, G, D, S)$ and $\alpha = \alpha(G, D, S)$, $0 < \alpha < 1$, such that,

$$|u| < Km^{\alpha} \tag{1}$$

throughout S. For general domains G, K and α cannot be given explicitly in any usable form. However, if G is the unit disc, $|z| < 1$, then simple explicit formulas for K and α have been given by Keith Miller [5]. In this paper an explicit formula of the type (1) will be derived for a certain class of star-shaped regions via the result of Miller and a simple estimation of the Riemann mapping function.

Let G be a simply connected domain which is star-shaped with respect to the origin. Let C denote the boundary of G. For any point z_0 on C, consider the circle $|z - z_0| = |z_0|$ which passes through the origin. Let Γ_{z_0} denote the sector of the circle $|z - z_0| = |z_0|$ with vertex at z_0, angular opening of $\gamma\pi$ at z_0, and the line segment connecting z_0 to the origin as the angle bisector. Assume that Γ_{z_0} is contained in G for all z_0 on C, which means that G satisfies a strong cone property.

Definition. Let $\Omega\gamma$ denote the class of simply connected domains G which are star-shaped with respect to the origin and which satisfy the strong cone property described in the previous paragraph.

Lemma. *Let G belong to $\Omega\gamma$ and let $\zeta = f(z)$ denote the Riemann mapping function that maps G onto $|\zeta| < 1$ such that $f(0) = 0$ and $f'(0) > 0$.*
If

$$\delta = \operatorname{dist}(z, C), \tag{2}$$

then

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