AN INTEGRAL SOLUTION OF AN ULTRAHYPERBOLIC EQUATION

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Introduction. The main purpose of this paper is to show that the 6-dimensional ultrahyperbolic equation

$$L[u] \equiv \sum_{i=1}^{3} \frac{\partial^2 u}{\partial x_i^2} - \sum_{i=1}^{3} \frac{\partial^2 u}{\partial y_i^2} = 0 \tag{0.1}$$

$$(u = u(x, y) = u(x_1, x_2, x_3, y_1, y_2, y_3))$$

has the integral solution

$$u(x, y) = \int_0^\infty t^2 \left[\iint_{\partial\Omega_1} d\omega_1 \iint_{\partial\Omega_2} g(x + t\omega_1, y + t\omega_2)\, d\omega_2 \right] dt. \tag{0.2}$$

The notation of (0.2) has the following interpretation. $\partial\Omega_i$ $(i = 1, 2)$ is the surface of the 3-dimensional unit sphere Ω_i; $d\omega_i$ is the element of surface area of $\partial\Omega_i$; ω_i is the unit vector from the center of Ω_i to a variable point on $\partial\Omega_i$; $x + t\omega_1$ and $y + t\omega_2$ are vectors in the 3-dimensional cartesian x-space and y-space. The function g is arbitrary but is assumed to possess the following three properties. (1) g is everywhere defined on (x, y)-space and is twice continuously differentiable. (2) On any bounded region of (x, y)-space, the integrals $(0 \le k = l + m \le 2)$

$$\int_0^\infty t^2 \left[\iint_{\partial\Omega_1} d\omega_1 \iint_{\partial\Omega_2} \left| \frac{\partial^k g(x + t\omega_1, y + t\omega_2)}{\partial x_i^l\, \partial y_i^m} \right| d\omega_2 \right] dt \tag{0.3}$$

are uniformly convergent. (3) For any (x, y) and uniformly in ω_1 and ω_2,

$$\lim_{t\to\infty} t^2 \left[\frac{\partial g(x + r\omega_1, y + s\omega_2)}{\partial r} - \frac{\partial g(x + r\omega_1, y + s\omega_2)}{\partial s} \right]_{r=s=t} = 0. \tag{0.4}$$

It is evident that (2) and (3) hold if g satisfies (1) and vanishes outside of a large sphere (i.e., has compact support).

The heuristic derivation of (0.2), given in §1, is similar to the method used in [2] to integrate the 3-dimensional wave-equation. In F. John's [3] investigation of the 4-dimensional ultrahyperbolic equation, integrals along lines were of special importance; the structure of (0.2) indicates their importance for the equation (0.1). The verification that (0.2) is indeed a solution of (0.1) is given in section 2. The method of proof partially complements that used by H. Lewy [4] to establish a simultaneous generalization of L. Asgeirsson's mean

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