ENUMERATION OF SYMMETRIC MATRICES

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1. Introduction. Let $H(n, r)$ denote the number of $n \times n$ matrices $[a_{ij}]$ where the a_{ij} are non-negative integers that satisfy

$$\sum_{i=1}^{n} a_{ij} = r = \sum_{j=1}^{n} a_{ij}, \qquad 1 \leq i, \quad j \leq n. \tag{1.1}$$

Anand, Dumir and Gupta [1] conjectured that for a given n and any r,

$$H(n, r) = \sum_{t=0}^{\binom{n-1}{2}} c_t \binom{r+n+t-1}{n+2t-1}, \tag{1.2}$$

where the c_t depend on n alone. This would imply that

$$\sum_{r=0}^{\infty} H(n, r)x^r = (1-x)^{-(n-1)^2-1}\psi(x), \tag{1.3}$$

where $\psi(x)$ is a symmetric polynomial in x of degree $(n-1)(n-2)$. It appears that the coefficients in $\psi(x)$ are positive integers. In particular, we have

$$\sum_{r=0}^{\infty} H(1, r)x^r = (1-x)^{-1},$$

$$\sum_{r=0}^{\infty} H(2, r)x^r = (1-x)^{-2},$$

$$\sum_{r=0}^{\infty} H(3, r)x^r = (1-x)^{-5}(1+x+x^2)$$

and probably

$$\sum_{r=0}^{\infty} H(4, r)x^r = (1-x)^{-10}(1+14x+87x^2+148x^3+87x^4+14x^5+x^6).$$

Carlitz [2] has considered the analogous problem for symmetric matrices. Here, we shall be concerned with the case $r = 2$, not considered by Carlitz.

2. Let $S(n)$ denote the number of $n \times n$ symmetric matrices $[a_{ij}]$, where the a_{ij} satisfy

$$a_{ij} = a_{ji} = 0, 1 \text{ or } 2; \qquad 1 \leq i, j \leq n;$$

and

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