SPECTRA OF MULTIPLIERS ON D_α

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1. Introduction. In this paper we shall discuss the spectral properties of multipliers on the Hilbert space D_α.

Fix H a Hilbert space with inner product $\langle\ \rangle$. By D_α, for α a fixed real number, we mean the Hilbert space of analytic vector valued functions, $f(z) = \sum_{n=0}^{\infty} a_n z^n$, such that $a_n \varepsilon H$ for $n = 0, 1, 2, \cdots$ and $\sum_{n=0}^{\infty} (n+1)^\alpha \|a_n\|_H^2 < \infty$. The inner product of D_α is given by $(f, g)_\alpha = \sum (n+1)^\alpha \langle a_n, b_n\rangle$ for $f(z) = \sum a_n z^n$ and $g(z) = \sum b_n z^n$ belonging to D_α (the absence of indices on the summation sign will henceforth imply the sum is to be taken from 0 to ∞). Note that each function of D_α is analytic in the open unit disc in the complex plane, $D_\alpha \subset D_\beta$ for $\alpha > \beta$ and λ_z^α mapping D_α into H defined by $\lambda_z^\alpha(f) = f(z)$ for each $f \varepsilon D_\alpha$ and $|z| < 1$ is a bounded linear transformation with norm $\|\lambda_z^\alpha\|^2 = \sum (n+1)^{-\alpha} |z|^{2n}$. A vector-valued function, $h(z)$, mapping the open unit disc into $\mathfrak{L}(H, H)$, the algebra of all bounded linear transformations mapping H into itself, is a multiplier from D_α to D_β if $h\cdot f \varepsilon D_\beta$ for each $f \varepsilon D_\alpha$ (where $h\cdot f$ denotes pointwise operation of h on f for z in the open unit disc). The set of all multipliers from D_α to D_β will be denoted by $\mathfrak{M}(D_\alpha, D_\beta)$. In this paper we will be primarily concerned with the case $\alpha = \beta$. A necessary condition for such a function to belong to $\mathfrak{M}(D_\alpha, D_\alpha)$ is that it be a bounded analytic vector-valued function mapping the open unit disc into $L(H, H)$. That is, $h \varepsilon \mathfrak{M}(D_\alpha, D_\alpha)$ implies $\sup_{|z|<1} \|h(z)\|_L < \infty$ and $h(z) = \sum A_n z^n$ where $A_n \varepsilon \mathfrak{L}(H, H)$ for $n = 0, 1, 2, \cdots$, and $\|\ \|_L$ denotes the norm of $\mathfrak{L}(H, H)$. Furthermore, the transformation T_h, $h \varepsilon \mathfrak{M}(D_\alpha, D_\alpha)$, mapping D_α into itself by $T_h(f) = h\cdot f$ for each $f \varepsilon D_\alpha$ is a bounded linear transformation. (For the proof of these statements and other results on multipliers of D_α see [3]). Finally, by $\mathfrak{L}(D_\alpha, D_\alpha)$ we mean the algebra of all bounded linear transformations mapping D_α) into itself.

2. We begin by giving a set which is always contained in the spectrum of a multiplier. Then under some additional hypotheses, we shall show that this set is exactly the spectrum. The following two lemmas will be needed in this development.

LEMMA 1. *$f \varepsilon D_\alpha$ if and only if $f' \varepsilon D_{\alpha-2}$ (where f' denotes the derivative of f with respect to z).*

LEMMA 2. *$h \varepsilon \mathfrak{M}(D_\alpha, D_\alpha)$ implies $h' \varepsilon \mathfrak{M}(D_\alpha, D_{\alpha-2})$.*

THEOREM 1. *If $h \varepsilon \mathfrak{M}(D_\alpha, D_\alpha)$ and T_h is invertible, then $(h(z))^{-1}$, denoted*

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