CLAMPED END BOUNDARY CONDITIONS FOR FOURTH-ORDER SELF-ADJOINT DIFFERENTIAL EQUATIONS

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Comparison theorems for oscillation in second-order linear equations have been derived by Walter Leighton [7] by use of a well-known theorem in the calculus of variations. In this paper we use a similar result to obtain comparison theorems for oscillation in fourth-order equations. Further use of this variational principle provides conditions on the coefficients of the fourth-order equation that imply oscillation.

Throughout r_1, r_2, q_1, q_2, q_3, r_1^*, r_2^*, q_1^*, q_2^* and q_3^* denote continuous real-valued functions on a ray $[a, \infty)$, and it is assumed that r_1, r_2, r_1^* and r_2^* are positive-valued. For sufficiently differentiable real functions y, we define the differential operators

$$D_1y = r_1y',$$

$$D_2y = r_2[(D_1y)' + q_1y],$$

$$D_3y = r_1[(D_2y)' + q_2D_1y]$$

and

$$Ly = (D_3y)' + q_1D_2y + q_3y. \tag{1}$$

Similarly, we define D_1^*, D_2^*, D_3^* and L^* by using the starred coefficients.

The operator L is the general fourth-order self-adjoint operator studied recently by Barrett [3]. The vector-matrix formulation of $Ly = 0$ is [3]

$$\begin{bmatrix} y \\ D_1y \\ D_2y \\ D_3y \end{bmatrix}' = \begin{bmatrix} 0 & 1/r_1 & 0 & 0 \\ -q_1 & 0 & 1/r_2 & 0 \\ 0 & -q_2 & 0 & 1/r_1 \\ -q_3 & 0 & -q_1 & 0 \end{bmatrix} \begin{bmatrix} y \\ D_1y \\ D_2y \\ D_3y \end{bmatrix}$$

from which the corresponding existence and uniqueness properties are obtained, with no differentiation of the coefficients.

Barrett [3] expresses the equation $Ly = 0$ in the vector-matrix system

$$\begin{cases} \alpha' = A\alpha + B\hat{\alpha} \\ \hat{\alpha}' = C\alpha - A^T\hat{\alpha} \end{cases} \tag{2}$$

where α, $\hat{\alpha}$, A, B and C are respectively

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