NEOFIELDS

By Lowell J. Paige

Introduction. The present paper is concerned with a particular generalization of a field. Specifically, we concern ourselves with an additive loop L whose non-zero elements form a multiplicative group G such that multiplication is both right and left distributive with respect to addition. The name *neofield* has been chosen to designate these generalized fields.

Our purpose is primarily to study the structure of neofields. The latter part of the paper is devoted to the study of *planar neofields* (neofields that may be used to coordinatize a projective plane in a manner to be explained later) and the application of the methods of neofields to allied geometrical problems.

The first chapter develops general results in the theory of neofields. Our primary concern is to determine the existence of a neofield N having an arbitrary predetermined group G as its multiplicative system; G is then called *admissible.* Toward this aim, a necessary and sufficient condition that a group G be admissible is given. We also show that the center of an admissible group is the multiplicative system of a sub-neofield $N' \subset N$. The problem of determining all admissible groups G is formulated and progress in the various phases is given.

In Chapter II, we devote our attention to abelian groups G and are able to show (a) all finite abelian groups are admissible, (b) the free abelian groups with n generators are admissible, (c) every infinite abelian group with n generators, provided G does not possess a unique element of order 2, is admissible. We then consider neofields for which the additive system is commutative or possesses the inverse property.

Defining a projective plane in Chapter III by means of homogeneous coordinates from a neofield S, we prove: "The additive loop of a finite planar neofield is a commutative, inverse property loop." This result is a significant necessary condition. Chapter IV gives generalizations of this result in connection with possibly more general finite geometries. The interest in finite geometries is motivated by the desire to answer the following question: "Given a finite projective plane geometry π, is it necessary that the number of points on a line be $p^n + 1$, p a prime?" No known counterexamples have ever been found. The present paper is, in the same sense, unsuccessful.

Finally Chapter V gives a necessary and sufficient condition that the multiplication table of a finite abelian group be orthogonal to a Latin square.

Many of the present investigations were undertaken as a thesis under the direction of R. H. Bruck and unquestionably some of his suggestions have been used without acknowledgement. The author takes this opportunity to thank Professor Bruck for his suggestions and aid.

Received February 17, 1948; in revised form, August 31, 1948.