SOLUTIONS OF SYSTEMS OF DIFFERENTIAL EQUATIONS IN TERMS OF INFINITE SERIES OF DEFINITE INTEGRALS

By Jesse Pierce

Introduction. The general solution of a differential equation of the first order and first degree can be found in terms of an infinite series of definite integrals.[1] The definite integrals appearing in the solution are solutions of linear differential equations.

The same method is applicable to finding the general solution of a system of differential equations. The functions giving the solution, however, are expressed in terms of infinite series of solutions of systems of linear differential equations. The general solution of each system of linear differential equations can be found in terms of infinite series of definite integrals.[2]

Hence the solution of the original system of differential equations is expressed in terms of infinite series of infinite series of definite integrals except in the case where the original system is linear or the system comprises just one differential equation of the first order.

In the present paper the general solution of a system of differential equations will be found in terms of infinite series of definite integrals in which every integrand consists of a finite number of terms.

The system of differential equations to be considered has the form

$$\frac{dy_i}{dt} = \varphi_i(t, y_1, \cdots, y_n) \qquad (i = 1, \cdots, n), \tag{1}$$

where the $\varphi_i(t, y_1, \cdots, y_n)$ are analytic in the y_j $(j = 1, \cdots, n)$ and have as coefficients $a_{i\mu_1\cdots\mu_n}$ functions of t which are integrable (Riemann) and satisfy the inequalities

$$| a_{i\mu_1\cdots\mu_n} | \leqq f(u), \tag{2}$$

$f(u)$ being a positive integrable function of u, the arc length of a rectifiable curve drawn from the origin to the point t. The exponent of y_j $(j = 1, \cdots, n)$ in the expanded form of the φ_i is represented by μ_j. The independent variable t is assumed to be of the form

$$t = \varphi(u) + \sqrt{-1}\,\psi(u), \tag{3}$$

where $\varphi(u)$, $\psi(u)$ are real functions with continuous first derivatives.

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[1] *Solutions of a differential equation of the first order and first degree in terms of infinite series of definite integrals*, presented by the author to the Ohio Section of the Mathematical Association of America, April, 1937.

[2] *Solutions of systems of linear differential equations in the vicinity of singular points*, American Mathematical Monthly, vol. 43 (1936), pp. 530–539.