ON MATRICES OF INTEGERS AND COMBINATORIAL TOPOLOGY

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1. **Introduction.** Our object is to give an elementary account of some algebraic theorems, with some immediate applications in combinatorial topology, in particular, in the theory of homology and cohomology[1] groups. The theorems are to a certain extent known, if in somewhat different forms.

The main tool in the algebraic part is the theory of group pairs, and in particular the question of when one group "resolves" or "completely resolves" another. The main theorems are on the existence of extensions of a homomorphism, and the existence of solutions of linear equations, with a matrix of integers and elements of an abelian group as unknowns. In each theorem, two types of conditions are employed, one using mod m properties, the other using group pairs. We shall use only discrete groups (except in Theorem 1).

The applications to topology are concerned with the relations between the ordinary homology theory, and the newer "dual" theory. An illustration of the convenience of the newer theory is given in Appendix I. For other illustrations, we mention the duality theorems (Kolmogoroff, Alexander, Čech, etc.), and properties of maps (see a following paper).

I. Group pairs and homomorphisms

2. **Group-pairs.** All groups will be abelian. 0 will denote the identity in any group. Let

$$mG = \text{all elements } mg,\ g \text{ in } G\ (m \text{ an integer}),$$

$$_mG = \text{all } g \text{ in } G \text{ such that } mg = 0.$$

Then $0G$ contains 0 alone. $G - G'$ is the difference (factor) group of G over the subgroup G'.

Let G, H, Z be three groups. If to each g in G and h in H there corresponds a $z = gh$ in Z, and both distributive laws are satisfied, we say G and H form a *group*

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[1] The relation of "coboundary" is becoming of increasing importance. Other terms have been used: dual boundary, upper boundary, inverse boundary, boundary in the dual subdivision, derived (of a function). The present term (accepted by E. Čech) offers definite advantages. In differential geometry, the (exact alternating) covariant tensors and the contravariant differentials play the same rôle as cocycles and (contra)cycles in the combinatorial theory; in fact, they may be obtained directly by a passage to the limit. Moreover, the prefix co is very convenient to handle.