ON LOCALLY-CONNECTED AND RELATED SETS

(Second paper)

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The subject matter of three recent papers by the author [1, 2, 3][1] has called forth remarks from Borsuk (on retracts), from Hurewicz (on fixed points) and corrections from Morse (on critical sets), which, together with some further developments induced thereby, we propose to consider in the present paper.

I. Chain-retraction

1. We shall need to refer in the sequel, explicitly and separately, to the following three characteristic properties relating a topological space $\mathfrak{R}$ to its *retract*, the closed set S (Borsuk [6]):

(a) there exists a single-valued transformation $T\colon \mathfrak{R} \rightarrow S$;

(b) T is continuous;

(c) $T = 1$ on S.

As a special case we might have for T a deformation over $\mathfrak{R}$ onto S leaving S point for point invariant. We should then call T a *deformation-retract*.

Once retracts are defined, the notions of AR, ANR follow. We have established in [1] the equivalences between types:

$$\text{ANR} \sim \text{LC}, \tag{1.1}$$

$$\text{AR} \sim \overline{\overline{\text{LC}}}, \tag{1.2}$$

where $\overline{\overline{\text{LC}}}$ designates in essence LC sets, in which in addition all spheres are homotopic to points. These two equivalences characterize absolute retracts by properties of local connectedness.

Now one of the chief features of our theory of chain-deformations [2] was the dissociation between the homotopic deformations of a set and its chains, and operations on the chains alone, regardless of what happens to the set itself. The degree to which this was accomplished there did not yield the extension of (1.1), (1.2) to HR sets. In truth we had not looked earnestly for it and were content in our paper to obtain certain other extensions of [1] from LC to HLC. A chance observation by Borsuk, to whom we mentioned this point, led us to the expected generalization as we shall now show. It will be profitable,

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[1] Numbers in square brackets refer to the bibliography at the end. The general notations and terminology are as in *Topology* [5]; the abridged notations are the same as in [1, 2]: LC = locally connected, H = homology, R = retract, NR = neighborhood retract, A in a compound abridged symbol stands for "absolute".