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SUBELLIPTIC ESTIMATES FOR THE $\bar{\partial}$-NEUMANN OPERATOR ON PIECEWISE SMOOTH STRICTLY PSEUDOCONVEX DOMAINS

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0. Introduction. Let Ω be a bounded pseudoconvex domain in $\mathbf{C}^n$ with the standard Hermitian metric. The $\bar{\partial}$-Neumann operator N is the inverse of the complex Laplacian $\Box = \bar{\partial}\bar{\partial}^* + \bar{\partial}^*\bar{\partial}$ on (p,q)-forms when $0 \leqslant p \leqslant n$ and $1 \leqslant q \leqslant n$. When Ω is smooth and strictly pseudoconvex, the existence and regularity of N were established by J. Kohn in [13] by proving that $\Box$ is subelliptic. Regularity results of N were also obtained for a wide class of other pseudoconvex domains with smooth boundaries (see Kohn [14], D. Catlin [4], and H. Boas and E. Straube [2]). On the other hand, recent results have shown that the $\bar{\partial}$-Neumann operator can be irregular on certain pseudoconvex domains with C^∞ boundaries (see D. Barrett [1] and M. Christ [6]).

Very little is known about N on domains with nonsmooth boundaries, even though, following Hörmander's L^2 existence theorem for $\bar{\partial}$, N is known to exist for any bounded pseudoconvex domain. The main difficulty lies in the fact that smooth forms are not necessarily dense in $\mathrm{Dom}(\bar{\partial}) \cap \mathrm{Dom}(\bar{\partial}^*)$ for the graph norm if the boundary is not smooth. Recently, G. Henkin and A. Iordan [8] established that N is compact on piecewise smooth strictly pseudoconvex domains using Bochner-Martinelli kernels. Theirs is the first result on nonsmooth pseudoconvex domains for N.

In this paper, we show that the $\bar{\partial}$-Neumann operator N still satisfies subelliptic estimates on a domain with piecewise smooth strictly pseudoconvex boundary. More precisely, we establish that N maps $L^2_{(p,q)}(\Omega)$ into $H^{1/2}_{(p,q)}(\Omega)$ when $1 \leqslant q \leqslant n-1$. (When $q = n$, it corresponds to the Dirichlet problem and one can easily see that N maps $L^2_{(p,n)}(\Omega)$ into $H^1_{(p,n)}(\Omega)$.) Moreover, we prove that $\bar{\partial}N$ and $\bar{\partial}^*N$ have the same properties. However, in contrast to smooth domains (see Kohn and L. Nirenberg [13]), subellipticity does not imply regularity in other Sobolev spaces in our case. This is due to the fact that the boundary conditions are overdetermined at the singularities of the boundary.

Our proof depends on the following observation: When Ω is piecewise strictly pseudoconvex, it can be approximated by a family of strictly pseudoconvex domains with smooth boundaries which are uniformly Lipschitz. We then prove a priori subelliptic estimates on each smooth strictly pseudoconvex domain with good control of the constants in each subdomain. Our approach for subellipticity

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