Hannan has written. With his retirement, I expect to see papers being generated even more rapidly.

ADDITIONAL REFERENCES

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Comment

R. Dahlhaus

This paper by Hannan is an excellent review of an important topic in time series analysis: the approximation of a nonparametric time series by a parametric rational one. The paper gives insight into the problems which arise and offers a variety of methods to tackle these problems.

To regard a fitted parametric model as an approximation to a nonparametric time series is clearly the correct point of view when dealing with parametric time series analysis. Many interesting papers dealing with related problems have been published in recent years and there is great need for further results to develop the theory sufficiently. The present paper is an important contribution to this goal.

It was therefore a pleasure for me to read this stimulating paper and to have been asked to comment on it. I will restrict my comments to the problem of estimation and in particular to the case of a one-dimensional process which is approximated by an autoregressive process.

1. THE APPROXIMATION CRITERION

Since the goal of the paper is the approximation of the transfer function, it seems to be natural to take a criterion which measures the quality of the approximation directly. Suppose the original series has an infinite autoregressive representation

$$\sum_{s=0}^{\infty} a_s Y_{t-s} = \varepsilon_t \quad \text{with } \varepsilon_t \text{ i.i.d.,}$$
$$E\varepsilon_t = 0, \quad \operatorname{var}(\varepsilon_t) = \sigma^2,$$

and Y_t is approximated by an AR(k)-process whose coefficients are estimated from the data by $\hat{a}_1(k), \cdots, \hat{a}_k(k)$, $\hat{\sigma}_k^2$. An appropriate approximation criterion then would be, for example,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{\sigma\hat{A}_k(\lambda)}{\hat{\sigma}_k A(\lambda)} - 1\right|^2 d\lambda, \tag{1}$$

where

$$A(\lambda) = \sum_{s=0}^{\infty} a_s \exp(-i\lambda s)$$

and

$$\hat{A}_k(\lambda) = \sum_{s=0}^{k} \hat{a}_s(k)\exp(-i\lambda s).$$

Considering the relative difference between $\hat{A}_k(\lambda)$ and $A(\lambda)$ is natural, since for Yule-Walker estimates (1) is approximately equal to $\sigma^{-2}T(\hat{a}(k) - a(k))' R(\hat{a}(k) - a(k))$ with $R = \{\operatorname{cov}(Y_i, Y_j)\}_{i,j}$, which tends weakly to a χ_k^2 distribution (if the true process Y_t is also an AR(k)-process), while the limit behavior of the absolute difference would depend on $A(\lambda)$. The choice of the $\mathscr{L}_2$ norm seems to be mainly for calculational convenience. However, by using the approximation $\log(\sigma/\hat{\sigma}_k) \approx (\sigma/\hat{\sigma}_k) - 1$ (or by adding the penalty term $2[(\sigma/\hat{\sigma}_k) - 1 - \log(\sigma/\hat{\sigma}_k)]$ for the innovation variance estimate to the criterion (1)) one

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