draw a curve of errors which, as a rule, will deviate very little from the original. All this, however, holds good only of the curves of presumptive errors. With the actual ones we cannot operate in this way, and the transition from the latter to the former seems in the meantime to depend on the eye's sense of beauty.

V. FUNCTIONAL LAWS OF ERRORS,

§ 17. Laws of errors may be represented in such a way that the frequency of the results of repetitions is stated as a mathematical function of the number, or numbers, expressing the results. This method only differs from that of curves of errors in the circumstance that the curve which represents the errors has been replaced by its mathematical formula; the relationship is so close that it is difficult, when we speak of these two methods, to maintain a strict distinction between them.

In former works on the theory of observations the functional law of errors is the principal instrument. Its source is mathematical speculation; we start from the properties which are considered essential in ideally good observations. From these the formula for the typical functional law of errors is deduced; and then it remains to determine how to make computations with observations in order to obtain the most favourable or most probable results.

Such investigations have been carried through with a high degree of refinement; but it must be regretted that in this way the real state of things is constantly disregarded. The study of the curves of actual errors and the functional forms of laws of actual errors have consequently been too much neglected.

The representation of functional laws of errors, whether laws of actual errors or laws of presumptive errors founded on these, must necessarily begin with a table of the results of repetitions, and be founded on interpolation of this table. We may here be content to study the cases in which the arguments (i. e. the results of the repetitions) proceed by constant differences, and the interpolated function, which gives the frequency of the argument, is considered as the functional law of errors. Here the only difficulty we encounter is that we cannot directly employ the usual Newtonian formula of interpolation, as this supposes that the function is an integral algebraic one, and gives infinite values for infinite arguments, whether positive or negative, whereas here the frequency of these infinite arguments must be $= 0$. We must therefore employ some artifice, and an obvious one is to interpolate, not the frequency itself, y, but its reciprocal, $\frac{1}{y}$. This, however, turns out to be inapplicable; for $\frac{1}{y}$ will often become infinite for finite arguments, and will, at any rate, increase much faster than any integral function of low degree.