INVARIANTS AND COVARIANTS OF CERTAIN FREQUENCY CURVES

By

RICHMOND T. ZOCH

Introduction. After the most convenient type of equation $y = f(x, a, b, c \ldots.)$ has been selected and the parameters a, b, c, …., in the selected equation have been determined so that for a given set of values x_i $(i = 1, 2, \ldots. n)$, the computed values y_i $(i = 1, 2, \ldots n)$ agree as closely as possible or as closely as is consistent with the observed values Y_i $(i = 1, 2, \ldots n)$, it may be desirable to make one or more of the transformations: (1) move the origin, (2) use a different scale (unit of measure), (3) change the total frequency.

This paper discusses certain invariants and covariants of the above transformations which were noted in developing the general theory for the Pearson Curves of frequency.

1. *Change of Origin.* Instead of considering the diff. eq.,

$$\frac{dy}{dx} = \frac{y(x-P)}{b_2 x^2 + b_1 x + b_0} \tag{1}$$

which is the diff. eq. from which the Pearson curves are derived, we take the more general diff. eq.,

$$\frac{dy}{dx} = \frac{y\,(x-P)}{b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0}. \tag{2}$$

Equation (1) is a special case of equation (2).

Make the following substitutions:

$$\left\{\begin{array}{ll} x = X + P, & b_n = B_n, \\ & nPb_n + b_{n-1} = B_{n-1}, \\ \frac{n(n-1)}{2!} P^2 b_n + (n-1) P b_{n-1} + & b_{n-2} = B_{n-2}, \\ \text{- - - - - - - - - - - -} & \\ P^n b_n + P^{n-1} b_{n-1} + \cdots\cdots & + b_0 = B_0, \end{array}\right. \tag{3}$$