ABSTRACTS OF PAPERS

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A Geometric Derivation of Fisher's z-transformation. J. B. COLEMAN, University of South Carolina.

In fitting points in a plane by a line so that the sum of the squares of the perpendicular deviations shall be a minimum, a second line is found for which the sum of the squares of the deviations is a maximum. Let Σd^2 be the sum of the squares of the deviations of the points from the minimum line, and ΣD^2 be the sum of the squares from the maximum line. Then $\Sigma D^2/\Sigma d^2 = (1 + r)/(1 - r)$. $\frac{1}{2} \log (1 + r)/(1 - r)$ is Fisher's z-transformation for testing the coefficient of correlation.

Large Sample Distribution of the Likelihood Ratio. ABRAHAM WALD, Columbia University.

The large sample distribution of the likelihood ratio has been derived by S. S. Wilks (*Annals of Math. Stat.*, Vol. 9 (1938)) in case of a linear composite hypothesis and under the assumption that the hypothesis to be tested is true. Here a general composite hypothesis is considered and the distribution in question is derived also in case that the hypothesis to be tested is not true. Let $f(x_1, \cdots, x_p, \theta_1, \cdots, \theta_k)$ be the joint probability density function of the variates $x_1, \cdots, x_p$ involving k unknown parameters $\theta_1, \cdots, \theta_k$. Denote by H_ω the hypothesis that the true parameter point $\theta = (\theta_1, \cdots, \theta_k)$ satisfies the equations $\xi_1(\theta) = \cdots = \xi_r(\theta) = 0$, $(r \leq k)$. Denote by λ_n the likelihood ratio statistic for testing H_ω on the basis of n independent observations on $x_1, \cdots, x_p$. For any parameter point θ let $\xi_{ij}(\theta) = \dfrac{\partial \xi_i(\theta)}{\partial \theta_j}$ and let $c_{ij}(\theta)$ be the expected value of $\dfrac{\partial \log f(x_1, \cdots, x_p, \theta)}{\partial \theta_i} \cdot \dfrac{\partial \log f(x_1, \cdots, x_p, \theta)}{\partial \theta_j}$ calculated under the assumption that θ is the true parameter point. For any θ denote by $A(\theta)$ the matrix $\| \xi_{ij}(\theta) \|$ $(i = 1, \cdots, r; j = 1, \cdots, k)$ and let $\| \sigma_{ij}(\theta) \| = \| c_{ij}(\theta) \|^{-1}$, $(i, j = 1, \cdots, k)$. Let furthermore $\| \sigma^*_{uv}(\theta) \|$, $(u, v = 1, \cdots, r)$ be the matrix equal to the product $A(\theta) \cdot \| \sigma_{ij}(\theta) \| \cdot \bar{A}(\theta)$, where $\bar{A}(\theta)$ is the transpose of $A(\theta)$. Finally let $\| \bar{c}^*_{uv}(\theta) \| = \| \sigma^*_{uv}(\theta) \|^{-1}$, $(u, v = 1, \cdots, r)$. For each n and θ denote by $y_{1n}(\theta), \cdots, y_{rn}(\theta)$ a set of r variates which have a joint normal distribution with mean values $\sqrt{n}\xi_1(\theta), \cdots, \sqrt{n}\xi_r(\theta)$ and covariance matrix $\| \sigma^*_{uv}(\theta) \|$, $(u, v = 1, \cdots, r)$. Denote the quadratic form $\sum_{v=1}^{r} \sum_{u=1}^{r} y_{un}(\theta) y_{vn}(\theta) \bar{c}^*_{uv}(\theta)$ by $Q_n(\theta)$. It has been shown that under certain assumptions on $f(x_1, \cdots, x_p, \theta)$, $\xi_1(\theta), \cdots, \xi_r(\theta)$ we have $\lim_{n=\infty} \{P(-2 \log \lambda_n < t \mid \theta) - P[Q_n(\theta) < t \mid \theta]\} = 0$ uniformly in t and θ, where for any z $P(z < t \mid \theta)$ denotes the probability that $z < t$ holds under the assumption that θ is the true parameter point. The distribution of $Q_n(\theta)$ is known and has been treated in the literature. If H_ω is true, then $\xi_1(\theta) = \cdots = \xi_r(\theta) = 0$, and $Q_n(\theta)$ has the χ^2 distribution with r degrees of freedom.

On the Integral Equation of Renewal Theory. W. FELLER, Brown University.

As is well-known, the equation $U(t) = G(t) + \int_0^t U(t - x)\, dF(x)$ has frequently been