

the definition of  $g$ , we have  $g'' < F(1 - F)x_0 - 4x_0f^2 < -2x_0f^2 < 0$ , which is impossible. Hence,  $g$  is nonnegative for all positive  $x$ , which completes the proof.

LEMMA 2.  $F(1 - F) \geq \pi f^2/2$  for  $0 \leq x < \infty$  with equality at 0 and  $\infty$ .

PROOF. Let  $h = F(1 - F) - \pi f^2/2$ . Then,

$$(2) \quad h' = f(1 - 2F) + \pi x f^2, \quad h'' = f^2(\pi - 2 - 2\pi x^2) - x f(1 - 2F).$$

It may be shown that  $h$  is continuous with derivatives of all order,  $h(0) = h(\infty) = 0$ ,  $h'(0) = 0$ , and  $h''(0) > 0$ . Let  $y_0$  be an extremum of  $h$ . Then, from (2)  $h'' = f^2(\pi - 2 - \pi y_0^2)$  at the point  $y_0$ . Hence,  $y_0 \leq (\pi - 2)^{1/2}$  if  $y_0$  is a minimum and  $y_0 \geq (\pi - 2)^{1/2}$  if  $y_0$  is a maximum, so that if a minimum and a maximum both exist, the minimum must precede the maximum. In view of this circumstance it is evident from the above mentioned properties of  $h$ ,  $h'$  and  $h''$  that a minimum cannot exist, and therefore that  $h$  is nonnegative for all positive  $x$ .

The results of Lemmas 1 and 2 can be rewritten respectively as

$$(3) \quad \left(F + \frac{f}{x} - \frac{1}{2}\right)^2 \geq \left(\frac{f}{x} - \frac{1}{2}\right)^2 + \frac{f}{x},$$

$$(4) \quad \left(F - \frac{1}{2}\right)^2 \leq \frac{1}{4} - \frac{\pi}{2} f^2.$$

For  $x \geq 0$  the upper bound of the theorem is obtainable from (3) and the lower bound from (4).

#### REFERENCES

- [1] R. D. GORDON, "Values of Mill's ratio of area to bounding ordinate of the normal probability integral for large values of the argument," *Ann. Math. Stat.*, Vol. 12 (1941), pp. 364-366.
- [2] Z. W. BIRNBAUM, "An inequality for Mill's ratio," *Ann. Math. Stat.*, Vol. 13 (1942), pp. 245-246.
- [3] W. FELLER, "An Introduction to Probability Theory and Its Application," John Wiley and Sons (1950), p. 145.

#### CORRECTION TO "SOME NONPARAMETRIC TESTS OF WHETHER THE LARGEST OBSERVATIONS OF A SET ARE TOO LARGE OR TOO SMALL"\*

BY JOHN E. WALSH

*U. S. Naval Ordnance Test Station, China Lake*

This note calls attention to the fact that Theorem 4 of this paper (*Annals of Math. Stat.*, Vol. 21 (1950), pp. 583-592) is only partially correct. The results  $\lim_{\Phi \rightarrow \infty} P_1(\Phi) = 0$  and  $\lim_{\Phi \rightarrow \infty} P_3(\Phi) = 1$  as well as the monotonicity properties

\* Received 1/29/52, revised form 9/19/52.