the sample size required to come to a terminal decision, but only certain aspects of it (for example, its expected value), can be handled as above, using the proper $W_{ijk}(x)$ at each stage.

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ON A CHARACTERISATION OF THE GAMMA DISTRIBUTION

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An intrinsic property of the gamma distribution, as proved by Pitman [1], is that if $X_1, X_2 \cdots, X_n$ are n identically distributed independent gamma variates with the distribution function

$$dF(X) = \frac{1}{\Gamma(p)} e^{-X} X^{p-1} \, dX \qquad (0 \leqq X \leqq \infty)$$

then the sum $X_1 + X_2 + \cdots + X_n$ is distributed independently of any function $g(X_1, X_2, \cdots, X_n)$ satisfying $g(X_1, X_2, \cdots, X_n) = g(\lambda X_1, \lambda X_2, \cdots, \lambda X_n)$ for any nonzero real λ. That is, $g(X_1, X_2, \cdots, X_n)$ should be a function independent of scale. In the present paper the converse theorem is proved for a particular class of g-function.

THEOREM. *Let $X_1, X_2, \cdots, X_n$ be n identically distributed independent random variables with a finite second moment. If the conditional expectation of the ratio of two quadratic forms $(\sum a_{ij} X_i X_j)/(\sum X_i)^2$, (where the elements of the matrix (a_{ij}) satisfy the relation $\sum a_{ii} \neq \sum a_{ij}/n$) for fixed sum $X_1 + X_2 + \cdots + X_n$ be equal to its unconditional expectation, then each X follows the gamma distribution.*

For a matrix $A = (a_{ij})$ where the relation $\sum a_{ii} = \sum a_{ij}/n$ holds, the method suggested does not lead to any solution of the problem. It is also interesting to note in this connection that the stronger assumption of stochastic independence of the sum $X_1 + X_2 + \cdots + X_n$ and $g(X_1, X_2, \cdots X_n)$ is not necessary for this particular class of g-function.

The following lemma is required for the proof of the Theorem.

LEMMA. *If u and v are two random variables such that for fixed v, the conditional expectation of $u/f(v)$, where $f(v)$ is a function of v, is equal to its unconditional expectation (provided it exists), then*

$$E(ue^{itv}) = E\{u/f(v)\} \cdot E\{f(v)e^{itv}\}.$$

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