Let X_{ni} be the characteristic function of the set A_{ni}. The sequence of random variables

$$X_{11}, X_{21}, X_{22}, X_{31}, \cdots$$

converges to 0 in probability but not a.s. so that (ii) implies (iii), completing the proof.

3. Proof of Theorem 2. To prove that (a) implies (b), assume that (a) is true and (b) is false. From Theorem A there exists a sequence A_n of events with $0 < P(A_n) \to 0$. Let X_n be the characteristic function of the set A_n. For all n, $f(X_n) \neq 0$ because if $f(X_{n_0}) = 0$, then by (a) the sequence of random variables, each of which is X_{n_0}, must converge to 0 in probability, contradicting $P(A_{n_0}) > 0$. By (a), $[f(X_n/f(X_n))] = 1$ for all n, so that the sequence of random variables $X_n/f(X_n)$ cannot converge to 0 in probability. However, it must, because $P(A_n) \to 0$. A contradiction has been reached, hence (a) implies (b).

Assuming (b) it is easy to show that $f(X) = E \mid X \mid$ is a norm on $\mathfrak{X}$ such that convergence in f is equivalent to convergence in probability. Theorem 2 is proved.

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DIVERGENT TIME HOMOGENEOUS BIRTH AND DEATH PROCESSES[1]

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1. Introduction. In a time-homogeneous birth and death process a population is considered, the size of which is given by the random variable $n(t)$ defined on the non-negative integers. If at time t the population size is n, the probability that a birth occurs in the time interval $(t, t + \Delta t)$ is $\lambda_n t + o(\Delta t)$; the probability of a death is $\mu_n t + o(\Delta t)$, and the probability of the occurrence of more than one

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