# DETERMINING SAMPLE SIZE FOR A SPECIFIED WIDTH CONFIDENCE INTERVAL

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**1. Introduction.** If an experimenter decides to use a confidence interval to locate a parameter, he is concerned with at least two things: (1) Does the interval contain the parameter? (2) How wide is the interval? In general the answer to these questions cannot be given with absolute certainty, but must be given with a probability statement. If we let $\alpha$ be the probability that the interval contains the parameter, and let $\beta^2$ be the probability that the width is less than $d$ units, then the general procedure is to fix $\alpha$ in advance and compute $\beta^2$. The value of $\beta^2$ is in general a function of the positive integer $n$, the sample size by which the confidence interval is computed. ($\beta^2$ is also a function of $\alpha$). In most confidence intervals, $\beta^2$ increases as $n$ increases. For any particular situation $\beta^2$ may be too low to be useful, hence an experimenter may wish to increase $\beta^2$ by taking more observations (increasing $n$). The problem the experimenter then faces is the determination of $n$ such that (A) the probability will be equal to $\alpha$ that the confidence interval contains the parameter, and (B) the probability will be equal to $\beta^2$ that the width of the confidence interval will be less than $d$ units (where $\alpha$, $\beta^2$, and $d$ are specified).

To solve this problem will generally require two things: (1) The form of the frequency function from which the sample of size $n$ is to be selected; (2) Some previous information on the unknown parameters in the frequency function.

This suggests that the sample be taken in two steps; the first sample will be used to determine the number of observations to be taken in the second sample so that (A) and (B) will be satisfied.

For a confidence interval on the mean of a normal population with unknown variance this problem has been solved by Stein [1] for $\beta^2 = 1$.

The purpose of this paper is to determine $n$, to satisfy (A) and (B) for distributions other than the normal.

**2. Theory.** Suppose $X$ is the width of a confidence interval on a parameter $\mu$ with confidence coefficient $\alpha$. Suppose further that it is desired that the probability be $\beta^2$ that $X$ be less than $d$. The problem is to determine $n$, the number of observations, on which to base $X$. Since $n$ depends on the random variables used in step one, $n$ is a random variable.

We will prove the following (we will use the notation $P(A)$ for the probability that the event A occurs):

Theorem. *Let the chance variable $X$ be the width of a confidence interval on a parameter $\mu$ based on a sample of size $n$. Suppose that $X$ depends on $n$ and on an unknown parameter $\theta$ '$\theta$ may be the parameter $\mu$). Suppose also that there exists a*

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