REFERENCE

[1] D. A. DARLING, "On a class of problems related to the random division of an interval," *Ann. Math. Stat.*, Vol. 24 (1953), pp. 239–253.

GENERALIZED D_n^+ STATISTICS[1]

BY A. P. DEMPSTER

Harvard University

1. Introduction. The purpose here is to present simplified derivation methods which can be applied to generalizations of some distributions derived by Birnbaum and Tingey [1] and Birnbaum and Pyke [2]. In the case of [1] the generalization is explicitly written down as equation (5)′. Other authors have noticed this generalization; it appears implicitly in equation (31) of Chapman [3] and is given explicitly by Pyke [4]. However the derivation given in the following section differs from the methods of other authors and gives a probabilistic meaning to each term in the summation formula (5)′. In the case of [2] explicit formulas are given for a special case of our generalization different from that considered by Birnbaum and Pyke.

Consider a sample of n from the uniform distribution on (0, 1). Denote the sample c.d.f. by $F_n(x)$. The relevant part of the curve $y = F_n(x)$ is entirely contained by the closed unit square $0 \leqq x \leqq 1$ and $0 \leqq y \leqq 1$, and within this square the population c.d.f. is represented by the line $y = x$. For $0 \leqq \delta < 1$ and $0 < \epsilon < 1$ the line joining $(0, \delta)$ and $(1 - \epsilon, 1)$ will be referred to as barrier (δ, ϵ). A set of such barriers moving away from $y = x$ may be conceived of, and we are concerned with a set of probabilistic questions about which barriers are crossed and where by the curve $y = F_n(x)$ as it passes from (1, 1) to (0, 0)

2. The basic derivation. Denote by $f_j (0 \leqq j \leqq n - 1)$ the probability that $y = F_n(x)$ crosses the barrier (δ, ϵ) at level $y = (n - j) / n$ not having crossed it at any level $y = (n - i) / n$ for $i < j$. Denote the abscissa of the intersection of the barrier (δ, ϵ) and level $y = (n - j) / n$ by m_j. Then it is easily checked that

$$m_j = \frac{1 - \epsilon}{1 - \delta}\left(1 - \delta - \frac{j}{n}\right). \tag{1}$$

Finally, let us use $b(r, s, p)$ for the binomial probability $\binom{s}{r} p^r (1 - p)^{s-r}$

An expression for f_j may be derived as follows. Given that $y = F_n(x)$ passes

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