SOME CONVERGENCE THEOREMS FOR STATIONARY STOCHASTIC PROCESSES[1]

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1. Introduction. Let $\mathcal{E}(t)$ $(-\infty < t < \infty)$ be a continuous stationary process of the second order (in the wide sense) with mean zero; that is,

$$E\mathcal{E}(t+u)\mathcal{E}(t) = \rho(u) \tag{1.1}$$

is a continuous function of u only, and

$$E\mathcal{E}(t) = 0, \qquad -\infty < t < \infty. \tag{1.2}$$

Here E means the expectation of a random variable.

We have, then,

$$\mathcal{E}(t) = \int_{-\infty}^{\infty} e^{it\lambda}\, dZ(\lambda), \tag{1.3}$$

and

$$\rho(u) = \int_{-\infty}^{\infty} e^{iu\lambda}\, dF(\lambda), \tag{1.4}$$

where $F(\lambda)$ is a bounded non-decreasing function such that

$$F(+\infty) - F(-\infty) = \rho(0) = E\,|\,\mathcal{E}(t)|^2,$$

and $Z(\lambda)$ is an orthogonal process such that

$$E\,|\,Z(\lambda') - Z(\lambda)|^2 = F(\lambda' - 0) - F(\lambda - 0). \tag{1.5}$$

$F(u)$ and $Z(\lambda)$ are called the spectral function and the random spectral function of $\mathcal{E}(t)$ respectively. (See, e.g., Doob [5], Chapter XI). Let

$$X(t) = f(t) + \mathcal{E}(t), \qquad -\infty < t < \infty, \tag{1.6}$$

where $f(t)$ is a numerical valued function, and consider

$$\int_{-\infty}^{\infty} x(t-s)K(s,n)\, ds, \tag{1.7}$$

$K(s, n)$ being also a numerical valued function depending on a parameter n.

Integrals of the type (1.7) appear in many fields in the theory of probability and statistics. For instance, we often encounter (1.6) in the problem of smoothing data of observed values, in the problem of predicting future values of $x(t)$, and in the problem of estimating the spectral density of a stationary process.

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