

THE NON-EXISTENCE OF CERTAIN PBIB DESIGNS

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1. Introduction. Let N be a Partially Balanced Incomplete Block (PBIB) design, (cf. Bose and Shimamoto, [1]), with three associate classes and with parameters

$$(1.1) \quad v, b, r, k, n_i, \lambda_i, p_{ju}^i; \quad (i, j, u = 1, 2, 3).$$

These parameters are not all independent but they are connected by the equations

$$(1.2) \quad \begin{aligned} bk &= vr; & \sum_{i=1}^3 n_i &= v - 1; & \sum_{i=1}^3 n_i \lambda_i &= r(k - 1); \\ p_{ju}^i &= p_{uj}^i; & n_i p_{ju}^i &= n_j p_{iu}^j = n_u p_{ij}^u; \\ \sum_{u=1}^3 p_{ju}^i &= n_j - \delta_{ij} & & (i, j, u = 1, 2, 3); \end{aligned}$$

where $\delta_{ij} = 0$ or 1 according as $i \neq j$ or $i = j$ respectively. Additional relations among the parameters (1.1) can be derived if the association scheme of the v treatments of N is completely known. Suppose, for example, that the association scheme of the given design N is of the rectangular type; that is, let us suppose that

$$(1.3) \quad v = v_1 v_2 \quad (v_1, v_2 \geq 2),$$

and that the treatments $\theta_{ij} (i = 1, 2, \dots, v_1; j = 1, 2, \dots, v_2)$ of the design N can be arranged in the form of a $v_1 \times v_2$ rectangle

$$(1.4) \quad \begin{array}{c} \theta_{11}, \theta_{12}, \dots, \theta_{1v_2} \\ \theta_{21}, \theta_{22}, \dots, \theta_{2v_2} \\ \dots \dots \dots \dots \\ \theta_{v_1 1}, \theta_{v_1 2}, \dots, \theta_{v_1 v_2} \end{array}$$

so that the first associates of any treatment θ_{ij} are the other $v_2 - 1$ treatments in the i th row; its second associates are the other $v_1 - 1$ treatments in the j th column and the remaining $(v_1 - 1)(v_2 - 1)$ treatments are its third associates. For the design N with the association scheme (1.4) it then follows that the matrices (p_{ju}^i) are given by

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