ON NECESSARY CONDITIONS FOR THE EXISTENCE OF SOME SYMMETRICAL AND UNSYMMETRICAL TRIANGULAR PBIB DESIGNS AND BIB DESIGNS[1]

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1. Summary. Consider PBIB designs based on triangular association scheme with $v = n(n-1)/2, b = (n-1)(n-2)/2, k = n, r = n-2, \lambda_1 = 1, \lambda_2 = 2$. It is established here that a necessary condition for the existence of these PBIB designs is the existence of symmetrical triangular PBIB designs with $v = b = (n-1)(n-2)/2, r = k = n-2, \lambda_1 = 1, \lambda_2 = 2$. Atiqullah [1] showed that the same condition is necessary for the existence of BIB designs with $v = (n-1)(n-2)/2, b = n(n-1)/2, k = n-2, r = n, \lambda = 2$. It is shown further that for an infinite number of values for n this condition cannot be satisfied.

2. Introduction. It is well known that for the triangular PBIB designs with $v = (n-1)(n-2)/2$

$$|NN'| = \rho_0 \rho_1^{n-2} \rho_2^{(n-1)(n-4)/2} \tag{1}$$

where N denotes the incidence matrix of the design and the ρ's are the characteristic roots. If the design is symmetric $|NN'|$ has to be a perfect square. Ogava [3] showed that a necessary condition for $|NN'|$ to be a perfect square is that

$$O_p = (-1, \rho_1)_p^{(n-2)(n-3)/2}(\rho_1, n-1)_p(\rho_1, n-2)_p^{n-2}(-1, \rho_2)_p^{(n-1)(n-2)(n-3)(n-4)/8} \cdot (\rho_2, 2)_p(\rho_2, n-2)_p(\rho_2, n-3)_p^{n-2} = +1 \text{ for all primes } p,$$

where the expressions of the form $(\alpha, \beta)_p$ are the Hilbert symbols. It will be shown that, for an infinite number of values of n, $O_p = -1$.

3. Conditions for the existence of some PBIB designs.

Lemma 1. *If there exists a PBIB design based on a triangular association scheme with $v = n(n-1)/2, b = (n-1)(n-2)/2, k = n, r = n-2, \lambda_1 = 1, \lambda_2 = 2$, then each of the blocks contains exactly one pair of the $(n-1)(n-1)/2$ pairs of each of the $n-1$ mutually first associate varieties appearing in the same row of the association scheme.*

Proof. Assume without loss of generality that a row of the association scheme contains the varieties $1, 2, \cdots, n-1$. Suppose further that there is a block of the design which contains the varieties $1, 2, \cdots k, k > 2$. Then there must be $(n-3)k$ additional blocks each containing exactly one of the varieties 1 through k. Since each of the varieties has to appear $n-2$ times the minimum number of blocks to be added is equal to the sum of the integers from $n-2-k$ down to 1. This sum, added to the sum obtained from the blocks previously counted,

Received July 9, 1962.

[1] This work was partially supported by the National Science Foundation, Grant-18976.