A PROPERTY OF SOME SYMMETRIC TWO-STAGE SEQUENTIAL PROCEDURES[1]

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1. Introduction. Some statistical problems concerning a one-parameter family of distributions may be formulated in such a way that they reduce to deciding whether some equivalent parameter θ is positive or negative. In this paper, the problem is assumed to be already in the reduced form and the family of distributions to be exponential with a real parameter θ. If $F_\theta(x)$ is the distribution function corresponding to θ and $F_{-\theta}(x) = 1 - F_\theta(-x^-)$, then the family and the problem have some degree of symmetry. If, in addition, the loss through wrong decision is an even function of θ which is zero for $\theta = 0$, then the problem is symmetric and it is reasonable that such symmetry be reflected in any procedure used. If Θ is the parameter space and a prior distribution on Θ symmetric about zero is selected, the corresponding Bayes procedure will certainly have the required symmetry and indeed any Bayes procedure which is symmetric can be shown to be also Bayes with respect to a symmetric prior distribution. Under certain conditions Wald [2] has shown that Bayes procedures and their limits form an essentially complete class and therefore, if attention is to be restricted to symmetric procedures, it may be further restricted to those procedures which are Bayes solutions for symmetric prior distributions. In a sequential solution to a problem of this type, Chernoff [1] obtained results which suggested that the expected number of observations required had a maximum at $\theta = 0$, precisely where the possible losses are least. The present paper shows that the same is true of two-stage sequential solutions in a somewhat more general context.

2. Assumptions and definitions. Suppose that there are available real-valued observations $\{X_i\}$ which are independently and identically distributed with common distribution belonging to a one-parameter exponential family. Suppose that the natural parameter set contains a neighborhood of the origin so that all members of the family are absolutely continuous with respect to the distribution given by $\theta = 0$. Let the latter probability measure be denoted by μ. Then the distribution corresponding to θ will have density $g(\theta) \exp\{\theta x\}$ with respect to μ.

Assumption 2.1. *The measure μ is symmetric about the origin and is either continuous or discrete. Also* $\mu(\{0\}) \neq 1$.

For the proof of Lemma 4 a further property is required of μ which will in

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