ON THE PROBABILITY OF LARGE DEVIATIONS OF THE MEAN FOR RANDOM VARIABLES IN $D[0, 1]$[1]

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1. Introduction. Let (Ω, S, P) be a probability measure space. Let $X_1(\omega)$, $X_2(\omega), \cdots$ be a sequence of B-measurable random variables in $\mathfrak{X}$ which are inde-dependently and identically distributed with common distribution $\mu(\cdot)$. Here $\mathfrak{X}$ is a separable complete metric space and B is the class of all Borel subsets. Let $\mu(n, \omega, \cdot)$ be the empirical probability measure of $X_1(\omega), X_2(\omega), \cdots, X_n(\omega)$, namely, the probability measure that assigns masses $1/n$ at each of the points $X_1(\omega), X_2(\omega), \cdots, X_n(\omega)$.

Let $f(x)$ be any measurable real-valued function on $\mathfrak{X}$ with $\int \exp(sf(x))\mu(dx) < \infty$ for all s. It has been shown by Cramér [3], Chernoff [2], Bahadur and Ranga Rao [1], etc. that

$$(1)\quad (1/n)\log P\{\omega: |\textstyle\int f(x)\mu(n, \omega, dx) - \int f(x)\mu(dx)| \geqq \epsilon\} \to \log \rho(f, \epsilon)$$

where, if $E(g) = \int g(x)\mu(dx)$ for any function g,

$$(2)\quad \rho(f, \epsilon) = \max\{\inf_{s \geqq 0} \exp[-s\epsilon - sE(f)]E(e^{sf}), \inf_{s \leqq 0} \exp[s\epsilon - sE(f)]E(e^{sf})\}.$$

Now let $\mathfrak{X}$ be a separable Banach space and let

(i) $\int \exp(s\|x\|)\mu(dx) < \infty$ for all s and

(ii) $\int x^*(x)\mu(dx) = 0$ for each continuous linear functional x^* on $\mathfrak{X}$. Sethuraman [6] (Theorem 7) has shown that

$$(3)\quad 1/n \log P\{\omega: \|(1/n)(X_1(\omega) + \cdots + X_n(\omega)\| \geqq \epsilon\} \to \log \rho(\mathfrak{X}_1^*, \epsilon)$$

where $\mathfrak{X}_1^* = \{x^*: \|x^*\| = 1\}$ and for any collection, $\mathfrak{F}$, of functions $\rho(\mathfrak{F}, \epsilon) = \sup_{f \in \mathfrak{F}} \rho(f, \epsilon)$.

Now, let $\mathfrak{X}$ be the space $D[0, 1]$ of all real valued functions $x(t)$ on $[0, 1]$ with the properties

(i) $x(t - 0)$ and $x(t + 0)$ exist for $0 < t < 1$ and $x(t) = x(t + 0)$

(ii) $x(t)$ is continuous at $t = 0$ and $t = 1$.

We endow this space with the J_1-topology of Skorohod [7] and it becomes a separable complete metric space. (See Section 2 for more details.) Let $\|x\| = \sup_{0 \leqq t \leqq 1} |x(t)|$. The main result of this note, proved in Section 3, can now be stated.

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