THE DISTRIBUTION OF THE GENERALIZED VARIANCE

By O. P. Bagai[1]

Panjab University

1. Introduction. The generalized variance i.e. the determinant of the sample variance and covariance matrix is defined [10] to be a measure of the spread of observations. Let S be the sample variance and covariance matrix of order $(p \times p)$ with n_1 degrees of freedom (d.f.) and let $\Sigma(p \times p) = E(n_1 S)$. The hth moment of the det. $|A|(= |n_1 S|)$, in the central case is given by Wilks [10] and that, in the noncentral case, by Herz [8] in the form of Laguerre polynomials and also by Constantine [7] in the form of Gaussian hypergeometric function of the type ${}_1F_1$.

Let k_i^2 $(i = 1, 2, \cdots, p)$ be the real and non-negative roots of the determinantal equation

$$|T - k^2\Sigma| = 0$$

where T is the noncentrality matrix of S. Assuming $k_i^2 = 0$ $(i = 2, 3, \cdots, p)$ and $k_1^2 \neq 0$, Anderson [1] gives the hth moment of the det. $|A|$ in the noncentral linear case as follows:

$$E(|A|^h) = 2^{ph} \exp\left(-\tfrac{1}{2} k_1^2\right) \prod_{i=1}^{p-1} \frac{\Gamma[\frac{1}{2}(n_1 - i) + h]}{\Gamma[\frac{1}{2}(n_1 - i)]} \cdot \sum_{j=0}^{\infty} \left\{ \frac{k_1^{2j}}{2^j j!} \frac{\Gamma(\frac{1}{2} n_1 + j + h)}{\Gamma(\frac{1}{2} n_1 + j)} \right\}. \tag{1.1}$$

It has been found difficult to obtain the distribution of the det $|A|$ in the noncentral case by making use either of the hth moments given by Herz [8] or that of Constantine [7]. The determination of the distribution of the det. $|A|$ by taking its hth moment (as in (1.1)), in the noncentral linear case for various values of p, has been found easy. For $p = 2$, 3 and 4 the author [3], [5], has already determined the distribution of the det. $|A|$ both for central and noncentral linear cases. We list only their results for completeness. In Section 3 the distribution of the det. $|A|$ in the noncentral linear case for higher values of $p = 5(1)10$ has been found and put in the standard form of the generalized Gauss' hypergeometric series defined as

$$ {}_0F_t(; r_1, r_2, \cdots, r_t; a) = 1 + \frac{1}{r_1 r_2 \cdots r_t} \frac{a}{1!} + \frac{1}{r_1(r_1+1) r_2(r_2+1) \cdots r_t(r_t+1)} \frac{a^2}{2!} + \cdots. \tag{1.2}$$

Then to determine the distribution in the central case for the same values of p,

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[1] Presently a United Nations T. A. O. Expert with the government of Sierre Leone.