# SOME TESTS FOR THE INTRACLASS CORRELATION MODEL

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**1. Introduction.** Let $X^{(i)}$ $(i = 1, 2, \cdots, k)$ be independent normal random $p$-vectors with mean vectors $\mu_i$ and nonsingular covariance matrices $\Sigma_i$. The problems with which we are concerned here are related to the comparison of dispersion matrices $\Sigma_i$'s, when each dispersion matrix takes the intraclass correlation form, i.e.,

$$\Sigma_i = \sigma_i^2[(1 - \rho_i)I + \rho_i ee'], \tag{1}$$

where $e' = (1, 1, \cdots, 1)$, and $I$ is an identity matrix of order $p \times p$. For this model, the problem of comparing dispersion matrices $\Sigma_i$ reduces to that of comparing $\sigma_i$'s and $\rho_i$'s of $k$ populations. For some related results on this model, refer to Wilks [8], Geisser [1], Votaw [7], and Selliah [6].

**2. Problems.** With the help of Roy's union-intersection principle [4], we propose in this paper test procedures for the following problems:

(i) To test the hypothesis $H$: $\rho_1 = \cdots = \rho_k$; $\sigma_1 = \cdots = \sigma_k$, against the alternative $A$: $\rho_i \neq \rho_j$; $\sigma_i \neq \sigma_j$, $i, j = 1, 2, \cdots, k$, $i \neq j$.

(ii) To test the hypothesis $H$: $\rho_i = 0$, $i = 1, 2, \cdots, k$, against the alternative $A$: $\rho_i \neq 0$, $i = 1, 2, \cdots, k$.

(iii) To test the hypothesis $H$: $\rho_1 = \cdots = \rho_k$, against the alternative $A$: $\rho_i \neq \rho_j$, $i, j = 1, 2, \cdots, k$, $i \neq j$.

**3. Reduction to canonical form.** Let $S$, a $p \times p$ matrix, have the Wishart distribution with mean $n\Sigma$ and degrees of freedom $n$, $n = N - 1$, and let $X$ be a normal random $p$-vector with mean vector $\mu$ and covariance matrix $\Sigma$. In addition, let $S$ be independent of $X$. For the intraclass correlation model $\Sigma$, there exists an orthogonal matrix $\Gamma$ with first row $e'/p^{\frac{1}{2}}$ such that $\Gamma\Sigma\Gamma' = \text{diag}(\alpha, \beta, \cdots, \beta)$, where

$$\alpha = \sigma^2[1 + (p - 1)\rho], \qquad \beta = \sigma^2(1 - \rho). \tag{2}$$

Let $W = \Gamma S \Gamma'$. Then the pdf of $W$ is

$$p(W) = \text{Const.}\, \alpha^{-n/2}\beta^{-m/2}|W|^{(n-p-1)/2} \exp\{-\tfrac{1}{2}w_{11}/\alpha - \tfrac{1}{2}\textstyle\sum_{r=2}^{p} w_{rr}/\beta\}, \tag{3}$$

where $m = n(p - 1)$. Let $Z = \Gamma X$, $\eta = \Gamma\mu$. Then $z_1$ is $N(\eta_1, \alpha)$, and $z_r$ is $N(\eta_r, \beta)$, $r = 2, \cdots, p$. The $z_r$'s are independently distributed for all $r = 1, 2, \cdots, p$.

Following Olkin and Pratt [3], we can obtain statistics $u$ and $v$ sufficient for $\alpha$ and $\beta$ and distributed independently as $\alpha\chi_a^2$ and $\beta\chi_b^2$ where the degrees of freedom $a$ and $b$ depend on our knowledge of $\mu$. For $\mu$ completely unknown,

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