FIDUCIAL THEORY AND INVARIANT PREDICTION[1]

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1. Introduction. We are concerned with the prediction of future observations y and functions $\psi(y)$ given past observations x when the joint distribution of x and y given an unknown parameter ω satisfies certain invariance properties. Our results are analogous to those given in an earlier paper on estimation (Hora and Buehler (1966), hereafter referred to as HB-1), and therefore some details are omitted from proofs. An identity is given involving the expectation of invariant functions $H(x, y, \omega)$ with respect to a distribution which we call the joint fiducial distribution of y and ω given x. The identity is used to define "best" invariant predictors in terms of fiducial expectations, and to establish sufficient conditions for prediction limits obtained from the fiducial distribution of y to be prediction analogues of confidence limits. The relationship to consistency criteria for fiducial distributions is indicated.

For regression models, prediction analogues of confidence limits were discussed by Eisenhart (1939), and later in textbooks such as Mood (1950), Section 13.3. In the interest of simplicity the present paper does not include a regression structure, which would be a fairly straightforward extension.

Weiss (1955) gave a general method of determining "confidence sets" for future observations y using a sufficient statistic $T(x, y)$ for ω. Our construction in Section 7 below is similar to his; his method would apply in certain cases lacking our group structure, but the present method applies in some cases when his sufficient statistic is lacking.

Kitagawa (1951) considered estimators α_1^* and α_2^* of a parameter α, based respectively on a past and a future sample and considered the accuracy of prediction of α_2^*. Later Kitagawa (1957) gave a theory of fiducial prediction quite close in spirit to the present work, but depending heavily on the theory of exponential families of distributions and on sufficient statistics, which are not required in the present treatment. Kitagawa's (1957) statistic h in Definition 2.3 and in equations (4.01) and (4.02) is an ancillary function of two sufficient statistics, and plays the same role as the quantity $t^{-1}y$ in the Appendix below. Kudō (1956) applied Kitagawa's theory in obtaining the fiducial distribution of the maximum of a future sample from a normal population. This case falls within the present scope since $\psi(y) = \max(y_1, \cdots, y_m)$ is an invariantly predictable function, as defined below in Section 5.

The Bayesian analysis of the prediction problem has been discussed for example by Fisher (1956), Chapter 3, Section 2, and by Jeffreys (1961), Chapter 3.

In the present paper attention is restricted to continuous variates. Similar

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