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A NOTE ON SEQUENTIAL MULTIPLE DECISION PROCEDURES[1]

By Isaac Meilijson[2]

Hebrew University of Jerusalem

0. Introduction. We introduce a family of procedures for choosing one out of k decisions concerning the (unknown) mean of a normal distribution (with known variance). Sobel and Wald proposed in [7] a procedure for the case $k = 3$. Their procedure can be expressed as a composition of two SPRT's for testing simple hypotheses. We followed their way of reasoning, but applied it to Anderson's modification of the SPRT [1]. We show that Paulson's procedure [4] is of the form of the suggested procedures, but can be improved. More explicitly, the (sampling) continuation region of some of the suggested procedures are subsets of those of Paulson's. As a consequence, the number of observations required by any one of them is never greater than the sample size required by Paulson.

1. The problem. Let $a_1 < a_2 < a_3 < \cdots < a_{k-1}$ be real numbers. Denote $a_0 = -\infty$, $a_k = +\infty$.

Let X be a rv normally distributed with unit variance and unknown mean θ. We want to choose one of the k decisions

$$D_i : \theta \,\varepsilon\, (a_{i-1}, a_i], \qquad i = 1, 2, \cdots, k, \tag{1}$$

when the loss function for the decision D_i is defined as the indicator of the complement of the interval $(a_{i-1} - \frac{1}{2}\Delta, a_i + \frac{1}{2}\Delta)$, where Δ is a positive real number satisfying

$$\Delta \ll \min_{1 \leq i \leq k-2} (a_{i+1} - a_i). \tag{2}$$

(The interval $(a_i + \frac{1}{2}\Delta, a_{i+1} - \frac{1}{2}\Delta)$ will be called "nonindifference interval").

A "solution" to the problem is a sequential procedure δ satisfying for a preassigned number $\alpha \,\varepsilon\, (0, 1)$.

$$\sup_\theta E_\theta l(\delta(X), \theta) \leq \alpha. \tag{3}$$

The present work deals with a special kind of solution, that can be described as a partition of the (n, s_n) plane into $k + 1$ sets: one sampling continuation set and k decision sets. The boundaries of these sets are broken lines.

This kind of procedure was treated extensively by Gordon Simons [5], [6]. The procedure presented in the next chapter is in some sense a special case of his general model.

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[2] Now at the University of California, Berkeley.