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ON FINITE PRODUCTS OF POISSON-TYPE CHARACTERISTIC FUNCTIONS OF SEVERAL VARIABLES

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1. Introduction. A characteristic function f of the n variables $t = (t_1, \cdots, t_n)$ is a Poisson-type characteristic function if it is of the form

$$f(t) = \exp\{iP(t) + \textstyle\sum_\epsilon \lambda_{\epsilon_1,\cdots,\epsilon_n}(e^{i(\epsilon_1\alpha_1t_1+\cdots+\epsilon_n\alpha_nt_n)} - 1)\},$$

where P is a polynomial of degree one without constant term and with real coefficients, the λ are non-negative constants, $\alpha = (\alpha_1, \cdots, \alpha_n)$ is a real vector, $\epsilon_j = 0$ or $1 (j = 1, \cdots, n)$ and $\sum_\epsilon$ indicates the summation on the $2^n - 1$ values of $\epsilon = (\epsilon_1, \cdots, \epsilon_n)$ different from $(0, \cdots, 0)$.

Therefore, the product f of two Poisson-type characteristic functions is of the form

$$(1.1)\quad f(t) = \exp\{iP(t) + \textstyle\sum_\epsilon [\lambda_{\epsilon_1,\cdots,\epsilon_n}(e^{i(\epsilon_1\alpha_1t_1+\cdots+\epsilon_n\alpha_nt_n)} - 1) + \mu_{\epsilon_1,\cdots,\epsilon_n}(e^{i\,\epsilon_1\beta_1t_1+\cdots+\epsilon_n\beta_nt_n)} - 1)]\}$$

with evident conditions on P, $\alpha = (\alpha_1, \cdots, \alpha_n)$, $\beta = (\beta_1, \cdots, \beta_n)$ and the constants λ and μ. In the case $n = 2$, we modify the notations and write (1.1) in the form

$$(1.2)\quad f(t) = \exp\{iP(t) + \lambda_1(e^{i\alpha_1t_1} - 1) + \mu_1(e^{i\alpha_2t_2} - 1) + \nu_1(e^{i(\alpha_1t_1+\alpha_2t_2)} - 1) + \lambda_2(e^{i\beta_1t_1} - 1) + \mu_2(e^{i\beta_2t_2} - 1) + \nu_2(e^{i(\beta_1t_1+\beta_2t_2)} - 1)\}.$$

In the case of one variable, it is known since P. Lévy [3] that the product of two Poisson-type characteristic functions has no indecomposable factor (in the sense of the decomposition of characteristic functions). But in the case of two variables, it is not the same: There are products of two Poisson-type characteristic functions which have indecomposable factors as it is shown in Section 2. Nevertheless, it is possible to find simple conditions assuring that the product of two Poisson-type characteristic functions has no indecomposable factor as it is shown in Sections 3 and 4. Finally, in Section 5, we give some results on the finite product of Poisson-type characteristic functions.

2. A counter-example. Let f be the product of two Poisson-type characteristic functions defined by

$$f(t_1, t_2) = \exp\{\lambda_1(e^{it_1} - 1) + \mu_1(e^{2it_2} - 1) + \nu_1(e^{i(t_1+2t_2)} - 1) + \lambda_2(e^{2it_1} - 1) + \mu_2(e^{it_2} - 1) + \nu_2(e^{i(2t_1+t_2)} - 1)\},$$

where λ_j, μ_j, ν_j $(j = 1, 2)$ are all positive. Then f has an indecomposable factor.

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