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INADMISSIBILITY OF THE BEST INVARIANT TEST WHEN THE MOMENT IS INFINITE UNDER ONE OF THE HYPOTHESES

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1. Introduction. Let $(\mathcal{Y}, \mathcal{A}, \lambda_i)(i = 1, 2)$ be probability spaces. For each $i = 1, 2$ and $y \varepsilon \mathcal{Y}$ let $F_i(\cdot, y)$ be a distribution function on the real line R such that $F_i(\cdot, \cdot)$ is $\mathcal{B} \times \mathcal{A}$ measurable where $\mathcal{B}$ is the σ-field of all Borel subsets of the real line R. Assume the distribution of $(X, Y) \varepsilon R \times \mathcal{Y}$ for $\theta \varepsilon R$ and $i = 1, 2$ is given by usual extension of

$$P_{i\theta}((X, Y) \varepsilon C \times D) = \int_D d\lambda_i(y) \int_C F_i(dx - \theta, y)$$

to measurable subsets of $R \times \mathcal{Y}$.

Consider the problem of testing $H_1 : i = 1$ versus $H_2 : i = 2$. For any level of significance a best invariant test φ_0 is of the form

$$\begin{aligned} (1.1) \qquad \varphi_0(x, y) &= 1 \quad \text{if} \quad \frac{d\lambda_2}{d(\lambda_1 + \lambda_2)}(y) > c \\ &= 0 \quad \text{if} \quad \frac{d\lambda_2}{d(\lambda_1 + \lambda_2)}(y) < c. \end{aligned}$$

We restrict attention to the case that the $F_i(\cdot, y)$ are absolutely continuous with respect to Lebesgue measure for each $y \varepsilon \mathcal{Y}$ and $i = 1, 2$. Denote the density of $F_i(\cdot, y)$ with respect to Lebesgue measure by $f_i(\cdot, y)$.

Lehmann and Stein [1] have shown that if $E_{i0}|X| < \infty$ for $i = 1, 2$ and if

$$(1.2) \qquad \lambda_1\{y : \frac{d\lambda_2}{d(\lambda_1 + \lambda_2)}(y) = c\} = 0$$

then φ_0 is admissible. Condition (1.2) guarantees that φ_0 is the essentially unique best invariant test at some level. Perng [2; Sections 4 and 5] has given examples showing that, with either the moment condition or (1.2) violated, φ_0 may not be admissible. The purpose of this note is to improve Perng's example concerning the moment condition.

Perng has shown that given any $\delta > 0$ one can construct an example in which $E_{i0}|X|^\alpha$ is, for $i = 1, 2$ finite or infinite according as $\alpha < 1 - \delta$ or $\alpha \geqq 1 - \delta$ and for which φ_0 is inadmissible. His example satisfies (1.2). The present example, given in Section 2, also satisfies (1.2) but is such that $E_{10}|X|^\alpha$ is as in Perng's example while $E_{20}|X|^\alpha < \infty$ for all $\alpha > 0$. This suggests the intuitive idea that knowledge of X is useful when the distributions of X under H_1 and H_2 are very different.

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