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CONVERGENCE IN DISTRIBUTION OF STOCHASTIC INTEGRALS[1]

BY MARK BROWN

Cornell University

0. Introduction. In this paper, convergence in distribution of sequences of quadratic mean stochastic integrals is studied by developing and extending an elegant approach introduced by J. Sethuraman [14]. Sethuraman's contribution is essentially contained in Theorem 3.1.

A type of convergence of stochastic processes, linear law convergence is introduced. It entails convergence of finite dimensional distributions and a condition on the product moment kernels of the processes. This condition has several equivalent forms and is discussed in Section 3.

Linear law convergence is well suited for deriving convergence in distribution of sequences of random variables $\{W_n, n = 1, 2, \cdots\}$, where $W_n \in L^2\{X_n(t), t \in T\}$. The convergence in distribution is derived without a sample path analysis. In fact, the random variables under consideration may not be pathwise defined. For example $\{W_n\}$ may be a sequence of quadratic mean stochastic integrals with the pathwise integrals not existing. On the other hand many important pathwise defined functionals of a process $\{X\{t\}, t \in T\}$, are not members of $L^2\{X(t), t \in T\}$, and thus not suited to linear law analysis.

Section 1 and Section 2 contain preliminary material on reproducing kernel Hilbert spaces and quadratic mean stochastic integrals. In Section 3, linear law convergence is introduced, and its basic properties derived. Section 4 contains a method by which a sequence of finite collections of random variables may be embedded into a sequence of continuous time processes satisfying the kernel condition for linear law convergence. In Section 5 linear law convergence is related to weak convergence over L^2 and reproducing kernel Hilbert spaces. In Section 6 several applications are derived.

1. Reproducing kernel Hilbert spaces. Let $[X(t)\ t \in T]$ be a complex valued second order stochastic process with product moment kernel K, so that $K(s, t) = E(X(s)\overline{X(t)})$. Let $\tilde{L}^2(X)$ be the set of all finite linear combinations $\sum_{i=1}^m a_i X(t_i)$, and let $L^2(X)$ be the closure of $\tilde{L}^2(X)$ under quadratic mean distance. $L^2(X)$ is a Hilbert space with inner product $(Z_1, Z_2) = E(Z_1 \bar{Z}_2)$.

For each $Z \in L^2(X)$, let g_Z be a function over T defined by $g_Z(t) = E(Z\overline{X(t)})$. It is easy to show that the operator A defined by $A(Z) = g_Z$ is one to one. It follows that the set $\{g_Z, Z \in L^2(X)\}$ becomes a Hilbert space under the inner product $(g_{Z_1}, g_{Z_2}) = (Z_1, Z_2)_{L^2(X)}$. Call this Hilbert space H_K. It follows that $L^2(X)$ and H_K

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