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ERROR ESTIMATION FOR A LIMIT THEOREM FOR DEPENDENT RANDOM VARIABLES[1]

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1. Introduction. Let (X_{nk}), $k = 1, 2, \cdots k_n$; $n = 1, 2, \cdots$ be a system of random variables. We investigate a limit theorem of a type studied by Loève [3] and particularly an error estimate for this theorem. The method used in finding the error estimate in this case applies to finding estimates for several of the theorems of [3].

2. A convergence theorem for independent systems. Let each X_{nk} have mean μ_{nk} and variance σ_{nk}^2 which we shall assume exists. Let $S_n = \sum_{k=1}^{k_n} X_{nk}$ and $\sigma_n{}^2 = \sum_{k=1}^{k_n} \sigma_{nk}^2$. If for each n, $X_{n1}, X_{n2}, \cdots, X_{nk_n}$ are independent we say that (X_{nk}) is an *independent system*. We write $\mathscr{L}(S_n) \to \mathscr{L}(X)$ if $F_n(x)$, the distribution function of S_n, converges to $F(x)$, the distribution function of X, at each continuity point of X. We write $\mathscr{L}(X) = \mathscr{L}(Y)$ when X and Y have the same distribution.

It is well known that if a random variable is infinitely divisible and has finite variance it can be represented by the formula of Kolmogorov [2] with unique real constant c and bounded nondecreasing function $K(u)$ which is right continuous and $K(-\infty) = 0$. (Henceforth, we shall call a function with these properties a *Kolmogorov function*). Also it is known [2] that if (X_{nk}) is an independent system of random variables having finite variances and such that $(X_{nk} - \mu_{nk})$ is infinitesimal, then $\mathscr{L}(S_n) \to \mathscr{L}(X)$ if there is a Kolmogorov function $K(u)$ and a constant c such that as $n \to \infty$

$$\sum_{k=1}^{k_n} \int_{-\infty}^{u} x^2 \, dF_{nk}(x + \mu_{nk}) \to K(u) \quad \text{at continuity points of} \quad K(u), \tag{2.1}$$

$$\sum_{k=1}^{k_n} \int_{-\infty}^{\infty} x^2 \, dF_{nk}(x + \mu_{nk}) \to K(\infty), \tag{2.2}$$

$$\sum_{k=1}^{k_n} \mu_{nk} \to c, \tag{2.3}$$

where $\mathscr{L}(X)$ is the infinitely divisible distribution determined by $K(u)$ and c.

3. A convergence theorem for dependent systems. The following notation is the same as that used in [3] and [4] with the exception that distributions will be used in the usual sense (i.e. $F(\infty) = P(-\infty < X < \infty) = 1$) rather than in the more generalized sense of Loève (i.e. $F(\infty) \leqq 1$). We recall that

$$F'_{nk}(x) = P(X_{nk} \leqq x / \textstyle\sum_{j=1}^{k-1} X_{nj})$$

$$E'(X_{nk}) = E(X_{nk} / \textstyle\sum_{j=1}^{k-1} X_{nj})$$

$$\sigma'^2_{nk} = E((X_{nk} - E(X_{nk}))^2 / \textstyle\sum_{j=1}^{k-1} X_{nj}).$$

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