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A PROPERTY OF POISSON PROCESSES AND ITS APPLICATION TO MACROSCOPIC EQUILIBRIUM OF PARTICLE SYSTEMS

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0. Introduction. It was shown by Derman [2] page 545, that if one takes a denumerably infinite state transient Markov chain with stationary measure μ and:

(i) at time 0 inserts $A_i(0)$ particles into state i, $i = 1, 2, \cdots$, where the $A_i(0)$ are independent Poisson variables with parameters $EA_i = \mu_i$;

(ii) lets each particle change states according to the transition matrix of the Markov chain, the particles behaving independently of one another: then for any n, the set of variables $\{A_i^{(n)}\ i = 1, 2, \cdots\}$, where $A_i^{(n)}$ is the number of particles in state i at time n, are independent and Poisson distributed with $EA_i^{(n)} = \mu_i$.

Port [8] studied the above system and derived the independent Poisson property with common parameter, for several collections of random variables describing various aspects of the behavior of the particle system.

One way of viewing this system is to regard the initial spatial process as a non-homogeneous Poisson process over the integers, and each particle (point of the Poisson process) as being independently mapped by a random sequence valued function into the sequence of its future states. The particle system thus generates a counting process on a space whose points are countable sequences of integers. This counting process can be represented by the collection of sequences $\{g_i(T_i)\}$ where the T_i are the initial states of the particles and the $\{g_i\}$ are i.i.d. random mappings. It then follows from a result apparently due to Karlin [4] page 497, and discussed by the author [1], that the counting process generated by $\{g_i(T_i)\}$ is Poisson, and strictly stationary. This strict stationarity generalizes the first order stationary of Derman, and from it the results of Port follow.

In general if we take a Markov process with stationary transition probability, state space $(\mathscr{X}, \mathscr{C})$ and σ-finite stationary measure μ, and at time 0 start with a Poisson $(\mathscr{X}, \mathscr{C}, \mu)$ process and move each particle independently according to the transition law of the Markov process, then we obtain a Poisson process on the space of particle paths. The measure of this Poisson process is strictly stationary and coincides with the measure on path space generated by μ and the transition law of the Markov process. This strong equilibrium property is derived in Section 3. The main tool is a generalization of Karlin's result (Section 2).

1. Definitions. Let $(\mathscr{X}, \mathscr{C}, \mu)$ be a measure space and let η be a random non-negative integer-valued (including $+\infty$) set function on $(\mathscr{X}, \mathscr{C})$. Define η to be

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