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ABSORPTION PROBABILITIES FOR CERTAIN TWO-DIMENSIONAL RANDOM WALKS[1]

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1. Introduction. In Section 2 of this paper, we consider absorption of certain finite random walks on three boundaries amenable to a method of images for the plane. Each of the three boundaries determines a class of walks to which the method is applicable; specifically, in the case of a rectangle or a right isosceles triangle with sides oriented along the axes, walks involving unit steps in directions 0, $\pi/2$, π and $3\pi/2$ and walks involving steps of length $2^{\frac{1}{2}}$ in directions $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$; in addition, for the rectangle, walks involving steps of all eight types; in the case of an equilateral triangle, walks involving unit steps in directions 0, $2\pi/3$ and $4\pi/3$, walks involving unit steps in directions $\pi/3$, π and $5\pi/3$, and walks involving steps of all six types.

For each of these eight boundary-walk combinations, it is possible to compute certain "untied" and "tied" probabilities. The first of these is the probability $P_{\Gamma,n}$ of absorption under a uniform distribution over all paths consisting of n steps of the specified types. The second is the probability $P^e_{\Gamma,\alpha,n}$, under a uniform distribution over all paths consisting of n steps of the specified types and ending at a specified interior point e, that absorption occurs at or before the $[\alpha n]$th step, $0 < \alpha \leqq 1$. We note that the tied absorption probability for one of the eight above boundary-walk combinations has been derived in [3], by an argument less direct than that presented here, for the case $\alpha = 1$.

Limits of expressions derived in Section 2 provide asymptotic absorption probabilities not only for the few cases examined there, but also, through the invariance principle, for rather large classes of walks, both "untied" and "tied". These are detailed in Section 3 and Section 4. The invariance principle simultaneously provides probabilities of absorption of two-dimensional[2] untied and tied Wiener processes on cylinders with triangular base, and hence the corresponding distributions of the time to absorption.

2. A method of images for the plane. We first illustrate the derivation of a "tied" absorption probability in terms of the boundary-walk combination treated in [3], i.e., the (open) equilateral triangle Γ together with a walk involving unit steps in directions 0, $2\pi/3$ and $4\pi/3$ (in the terminology of [3], steps of type A, B and C). We are interested in first computing the probability $P^e_{\Gamma,\alpha,n}$ that an n-step walk of this type, starting at the origin and terminating at the interior point e of Γ, exits Γ

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[2] The qualifier "two-dimensional" refers here to the range of the process (i.e. space) rather than, as for example in [6], to the domain (i.e. time).