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Discussion of “Is Bayes Posterior just Quick and Dirty Confidence?” by D. A. S. Fraser

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1. CONFIDENCE REGION ESTIMATION

The author has written an interesting article on the relationship of confidence distribution and Bayesian posterior distribution. Confidence distribution has its origin from Fisher’s fiducial distribution, and in this discussion we refer to it simply as the “confidence distribution approach.” It allows frequentists to assign confidence intervals (or, more generally, confidence regions) to the outcome of estimation procedures.

The idea can be simply described as follows. Consider a statistical model with a family of distributions $p_\theta(y)$, where y is the observation and θ is the model parameter. We assume that the observed y is generated according to a true parameter θ_* which is unknown to the statistician. If we can find a real-valued quantity $U(y;\theta)$ that depends on θ and y such that for all θ, when y is generated from $p_\theta(y)$, $U(y;\theta)$ is uniformly distributed in $(0, 1)$, then we can estimate the confidence interval of θ given an observation y as the set $I_{\alpha,\beta}(y) = \{\theta : U(y;\theta) \in (\alpha, \beta)\}$ for some $0 \le \alpha \le \beta \le 1$. An interpretation of this confidence region is that no matter what is the true underlying θ_* that generates y, the region $I_{\alpha,\beta}(y)$ contains the true parameter θ_* with probability $\beta - \alpha$ (when y is generated according to θ_*).

Indeed, the above interpretation is a very natural definition of confidence region in the frequentist setting. It does not assume that θ_* is generated according to any prior, and the interpretation holds universally true for all possible θ_* in the model. This interpretation can be compared to a confidence region from the Bayesian posterior calculation that assumes that θ_* is generated according to a specific prior which has to be known to the statistician. If the statistician chooses the wrong prior, then the confidence region calculated from the Bayesian approach will be incorrect in that it may not contain the true parameter θ_* with the correct probability.

The paper takes this interpretation of confidence region, and goes on to provide several examples showing that the Bayesian approach does not lead to correct confidence estimates for all θ_*. The author then argued that the confidence distribution approach is the more “correct” method for obtaining confidence intervals and the Bayesian approach is just a quick and dirty approximation.

One question that needs to be addressed in the confidence distribution approach is how to construct a statistics $U(y_0;\theta)$ with the desired property. The author considered the quantity $U(y_0;\theta) = \int_{y \le y_0} p_\theta(y)\,dy$, which is well-defined if the observation y is a real-valued number. This corresponds to the proposal in Fisher’s fiducial distribution. The idea of fiducial distribution received a number of discussions throughout the years, and is known to be adequate for unconstrained location families (for which the fiducial confidence distribution matches the Bayesian confidence distribution using a flat prior). However, the general concept is controversial, and largely regarded as a major blunder by Fisher.

In this discussion article we will explain why the idea of confidence distribution with

$$U(y_0;\theta) = \int_{y \le y_0} p_\theta(y)\,dy$$

has not received more attention for general statistical estimation problems, although it does give confidence region estimates that fit the frequentist intuition.

2. SUBOPTIMALITY

The purpose of confidence distribution is to provide a confidence region that is consistent with the frequentist definition. However, one flaw of this approach is that the result it produces may not be optimal. While this issue was pointed out in the article, it was not explicitly discussed. In my opinion, this is the main reason why the idea of confidence distribution hasn’t become more popular in statistics. Therefore, this section provides a more detailed discussion on this issue.

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