Acta Math., 167 (1991), 127–151

Towards a Paley–Wiener theorem for semisimple symmetric spaces

by

P. DELORME and M. FLENSTED-JENSEN

Faculté de Sciences de Luminy
Marseille, France

The Royal Veterinary- and Agricultural University
Frederiksberg, Denmark

Introduction

In the later years several Paley–Wiener type theorems have been established for Fourier transforms on special classes of non-flat symmetric spaces. In particular this is so for the case of Riemannian symmetric spaces of the non-compact type, see e.g. Ehrenpreis–Mautner [10], Helgason [17] and Gangolli [16], and for the case of the semisimple non-compact Lie groups (which in their own right are non-Riemannian symmetric spaces), see e.g. Zelobenko [31], Ehrenpreis–Mautner [10], Campoli [5], Arthur [1], Delorme [8] and Clozel–Delorme [6] and [7].

For a general non-Riemannian semisimple symmetric space G/H, the question of how the Fourier transform should be defined and in particular how it should be normalized is not definitively clarified. However a fairly explicit Plancherel formula has been announced by Oshima and Sekiguchi. A Paley–Wiener type theorem should either refer to a specific normalization or it should be formulated independent of normalizations. In any case a Paley–Wiener theorem should characterize the image under the Fourier transform of natural classes of compactly supported functions or maybe classes of very rapidly decreasing functions.

The main result of this paper is Theorem 1, which exhibits a large class of functions as Fourier transforms of compactly supported K-finite C^∞-functions on G/H. The proof is in fact rather elementary. However it seems to us, in spite of this, that the content of the theorem is not uninteresting. To illustrate this we specialize in Theorem 2 to the case of a non-compact semisimple Lie group. Theorem 2 was first proved by Campoli [5] for the rank one case and in general by Arthur [1]. For them our Theorem 2 is a simple corollary of their Paley–Wiener theorem, which is rather difficult to prove. E.g. Harish-Chandra's Plancherel formula and the theory of differential equations with