Viscosity solutions of fully nonlinear second-order equations and optimal stochastic control in infinite dimensions. Part I: the case of bounded stochastic evolutions

by

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I. Introduction

We study here fully nonlinear second-order degenerate elliptic equations of the following form

$$F(D^2u, Du, u, x) = 0 \quad \text{in } H \tag{1}$$

where H is a separable Hilbert space, x denotes a generic point in H, u—the unknown—is a function from H into $\mathbf{R}$, Du and D^2u denote the first and second Fréchet differentials that we identify respectively with elements of H, and symmetric bounded bilinear forms over H or indifferently bounded symmetric operators on H. We will denote by $L'(H)$ the space of all symmetric bounded bilinear forms over H and we will always assume at least that

$$F \text{ is bounded, uniformly continuous on bounded sets of } L'(H)\times H\times\mathbf{R}\times H. \tag{2}$$