RANDOM WALK ON COUNTABLY INFINITE ABELIAN GROUPS

BY

H. KESTEN and F. SPITZER

Cornell University, Ithaca, N. Y., U.S.A.

1. Introduction

Given a probability measure μ on a countably infinite Abelian group $\mathfrak{G}$ we propose to study the properties of the potential kernels

$$\sum_{n=0}^{\infty} \mu^{(n)}(x) \quad \text{and} \quad \sum_{n=0}^{\infty} [\mu^{(n)}(0) - \mu^{(n)}(x)], \quad x \in \mathfrak{G}. \tag{1.1}$$

Here 0 is the identity element of the (additive) group $\mathfrak{G}$, $\mu^{(0)}$ is the probability measure all of whose mass is concentrated at 0, $\mu^{(1)} = \mu$ and $\mu^{(n)}$ is the n-fold convolution of μ with itself.

Roughly speaking, the purpose of this paper is to imitate and extend basic results in [10] (Chapter 7 and parts of earlier chapters). There the attention was strictly confined to the groups $\mathfrak{G} = Z_d$, the groups of d-dimensional integers, or lattice points in Euclidean space of dimension d. Thus the basic ideas, methods, and notation are exactly those in [10] when possible—and most of the difficulties which arise because $\mathfrak{G}$ is more complicated than Z_d can be overcome by the use of certain measures induced by the given measure μ on cyclic subgroups of $\mathfrak{G}$.

It will be assumed throughout that the measure μ is *aperiodic*, i.e. that the support of μ generates all of $\mathfrak{G}$. (Note however that $\mathfrak{G}$ must be infinite. When $\mathfrak{G}$ is finite everything we do is either trivial or well known but the results are by no means the same.) Given μ we define on $\mathfrak{G}$ the Markov process (random walk) X_n with transition function

$$P_x[X_1 = y] = P(x, y) = \mu(y - x),$$

$$P_x[X_n = y] = P_n(x, y) = \mu^{(n)}(y - x), \quad x, y \in \mathfrak{G}, \ n \geqslant 0.$$

Here $P_x[\cdot]$ is the probability measure induced by the joint probabilities for finite paths starting at $X_0 = x$, and the associated expectation will be denoted by $E_x[\cdot]$.