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On the Titchmarsh convolution theorem

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1. Introduction and statement of results

Let M be the set of all finite complex-valued Borel measures $\mu\neq 0$ on $\mathbf{R}$. Set

$$l(\mu)=\inf(\operatorname{supp}\mu).$$

The classical Titchmarsh convolution theorem (see, e.g. [4], Chapter VI.F, [6], §16.2) claims that if measures $\mu_1, \mu_2, \dots, \mu_n$ belong to M and satisfy

$$l(\mu_j)>-\infty, \quad j=1,2,\dots,n, \tag{1}$$

then

$$l(\mu_1*\mu_2*\dots*\mu_n)=l(\mu_1)+l(\mu_2)+\dots+l(\mu_n), \tag{2}$$

where '$*$' denotes the operation of convolution.

Simple examples show that condition (1) is essential. One may set

$$\mu_1=\sum_{m=0}^{\infty}\frac{\delta_{-km}}{m!}, \quad \mu_2=\sum_{m=0}^{\infty}(-1)^m\frac{\delta_{-km}}{m!}, \quad k>0, \tag{3}$$

where δ_x is the unit measure concentrated at the point x. Clearly, $\mu_1*\mu_2=\delta_0$. We see that $l(\mu_1)=l(\mu_2)=-\infty$ while $l(\mu_1*\mu_2)=0$.

It was Y. Domar [2] who first established that condition (1) can be replaced by a sufficiently fast decay of μ_j at $-\infty$: there exists $a>2$, such that

$$|\mu_j|((-\infty,x))=O(\exp(-|x|^a)), \quad x\to-\infty, \; j=1,2,\dots,n.$$

The best possible condition on decay of μ_j was obtained in [8].