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Polynomials on dual-isomorphic spaces

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In this note we study isomorphisms between spaces of polynomials on Banach spaces. Precisely, we are interested in the following question raised in [5]: If X and Y are Banach spaces such that their topological duals X' and Y' are isomorphic, does this imply that the corresponding spaces of homogeneous polynomials $\mathcal{P}(^nX)$ and $\mathcal{P}(^nY)$ are isomorphic for every $n \geq 1$?

Díaz and Dineen gave the following partial positive answer [5, Proposition 4]: Let X and Y be dual-isomorphic spaces; if X' has the Schur property and the approximation property, then $\mathcal{P}(^nX)$ and $\mathcal{P}(^nY)$ are isomorphic for every n. Observe that the Schur property of X' makes all bounded operators from X to X' (and also from Y to Y') compact. That hypothesis can be considerably relaxed. Following [6], [7], let us say that X is regular if every bounded operator $X \to X'$ is weakly compact. We prove the following result.

Theorem 1. *Let X and Y be dual-isomorphic spaces. If X is regular then $\mathcal{P}(^nX)$ and $\mathcal{P}(^nY)$ are isomorphic for every $n \geq 1$.*

In fact, it is even true that the corresponding spaces of holomorphic maps of bounded type $\mathcal{H}_b(X)$ and $\mathcal{H}_b(Y)$ are isomorphic Fréchet algebras. Observe that the approximation property plays no rôle in Theorem 1. This is relevant since, for instance, the space of all bounded operators on a Hilbert space is a regular space (as every C^*-algebra [7]) but lacks the approximation property.

Our techniques are quite different from those of [5] and depend on certain properties of the extension operators introduced by Nicodemi in [10]. For stable spaces (that is, for spaces isomorphic to its square) one has the following stronger result.

Theorem 2. *If X and Y are dual-isomorphic stable spaces, then $\mathcal{P}(^nX)$ and $\mathcal{P}(^nY)$ are isomorphic for every $n \geq 1$.*

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