NOTES & DISCUSSIONS

SOME THINGS JUST DON'T BELONG

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In *Logical Forms*, Sainsbury [*1991*, 219] remarks:

> Whatever one might want from the "Law of Identity" (the validity of every sentence of the form "$a = a$") can be obtained from a conditional version of the law (the validity of every sentence of the form "$(\exists x)(x = a) \rightarrow (a = a)$").

Sainsbury's restriction of the "Law of Identity" points the way to a novel solution for Russell's Paradox, obtained by restricting Cantor's Comprehension Principle, as in (1):

$$(1) \quad (\exists y)(\forall x)[(x \in y) \leftrightarrow ((x = x) \;\&\; (...x...))] .$$

From (1) we have (2):

$$(2) \quad (\forall x)[(x \in R) \leftrightarrow ((x = x) \;\&\; \sim(x \in x))] ;$$

and from (2), (3):

$$(3) \quad (R \in R) \leftrightarrow ((R = R) \;\&\; \sim(R \in R)) .$$

(3) and Sainsbury's principle together yield that there is no R:

$$(4) \qquad \sim (\exists x)\,(x = R)\,.$$

Of course, (1) is to no avail if it does not eliminate the other paradoxes. On the other hand, if (1) does — and if, as Sainsbury's restriction of the "Law of Identity" suggests, not everything *is* self-identical[1] — (1) is preferable to Zermelo's Axiom of Separation, for a fundamental reason: unlike Zermelo's axiom, it restricts set-membership *to* the self-identical.[2]

References

GREENBERG, William. 1995. *A theory of complexes*, to appear in Epistemologia, No. 2, 1995. Available from IPPE by ftp to: Phil-Preprints.L.Chiba-U.ac.jp/pub/Preprints/Logic/Greenberg.

SAINSBURY, Mark. 1991. *Logical forms*, Cambridge, Basil Blackwell.

[1] If everything *were* self-identical, why *restrict* the "Law of Identity"?

[2] The appropriateness of this restriction is established by (5):

$$(5) \qquad (\forall x)\,((\exists y)(x = y) \leftrightarrow (\exists y)(x \in y))\,.$$