Masayoshi Nagata (1927–2008) and his mathematics

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Masayoshi Nagata, professor emeritus of Kyoto University, died of cancer in Kyoto on August 27, 2008, at the age of 81 years. He was born in the city of Ohbu near Nagoya and graduated from Nagoya Imperial University in 1950. He was a student of Tadasi Nakayama and published his first research articles in *Nagoya Mathematical Journal* while he was an undergraduate student. He became a research assistant in the Faculty of Science of Nagoya University the same year he graduated. In 1953, he moved to Kyoto University as an instructor. Many young people talented in algebra and algebraic geometry gathered together around Yasuo Akizuki. Nagata became an associate professor of Kyoto University in 1957 and was promoted to professor in 1963, succeeding Yasuo Akizuki as Chair of Algebra. He held this professorship at Kyoto University until his retirement in 1990.

The mathematical influence of Masayoshi Nagata is enormous not only through his research works but also through his contributions to the domestic and international mathematical communities. He played a quite active role in the mathematical community in Japan by serving as trustee of the Mathematical Society of Japan and as a member of the Science Council of Japan. At the International Mathematical Union, he served as a member of the Executive Committee between 1975 and 1978 and as vice president from 1979 to 1982. He was awarded the Chunichi Cultural Prize in 1961, the Matsunaga Prize in 1970, and the Japan Academy Prize in 1986. The Order of the Sacred Treasure, Gold and Silver Star, was conferred on him in November 1998.

Nagata was an outstanding mathematician, exceptionally talented in looking into the intrinsic nature of problems and expressing his insights through counterexamples. He was nicknamed "Mr. Counterexample" in admiration. He was a good teacher as well and wrote many textbooks including the introductory ones listed at the end of this article. Let us now trace his mathematical achievements in commutative algebras and algebraic geometry.

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1. Ring theory

Nagata's earliest article is a joint work with Noboru Ito [1] in which they studied complete groups, that is, groups G with trivial center $Z(G) = \{e\}$ and $\mathrm{Aut}(G) = \mathrm{Inn}(G)$. In addition to this article, he wrote another article [9] in group theory treating groups with an involution σ such that $\sigma(g) = g$ implies $g = e$. His interest in groups and applications to invariant theory is seen frequently throughout his mathematical career.

The works that immortalize the name of Nagata begin with those on local rings. Predecessors in this field are W. Krull, C. Chevalley, I. S. Cohen, O. Zariski, P. Samuel, and others. In fact, S. S. Abhyankar called Nagata the real successor of Krull. Furthermore, when he began research at Nagoya University as an undergraduate student, T. Nakayama and G. Azumaya were among the teachers at the Department of Mathematics. Nagata's research on local rings was more exhaustive than that of these predecessors.

Nagata's first work on local rings was on a generalization of Cohen's theorem on the structure of complete local rings, where he weakened the Noetherian assumption on the ring in showing the existence of the coefficient ring (see [2], [10]). His earlier works were related to complete semilocal rings, rings of quotients, and various radicals including the Jacobson radicals (see [3], [4], [15], [5]). These works led Nagata to the notion of the Henselization of a local ring which was first introduced in [11], [16], and [40]. Grothendieck and Dieudonné [R3] later treated the notion categorically in terms of schemes. In [14], the notion of Weierstrass rings was introduced as an application of Henselian regular local rings. His research expanded to the integral closure (the derived normal ring) of Noetherian integral domains (see [21]), the analytic irreducibility (the completion is an integral domain), and the analytic unramifiedness (the nonexistence of nilpotent elements in the completion; see [20]). In his works on derived normal rings, one can see the influences of Y. Akizuki and Y. Mori at Kyoto University, where Nagata moved about that time from Nagoya University. In the work on the completions, one can see the influences of C. Chevalley and O. Zariski. Nagata produced famous counterexamples for analytic irreducibility (see [37]) and analytic unramifiedness (see [20]). In [19], basic results were formulated in the modern language used in recent textbooks on commutative algebra. Nagata gave in [29] a criterion for a Noetherian integral domain R to be a unique factorization domain in terms of a prime element x of R and the factoriality of the ring of quotients $R[x^{-1}]$, which was taken up by Samuel [R11] in his Tata lecture notes on unique factorization domains.

Nagata had geometric inclinations in considering the problems on local rings. One of the problems that led him to the important geometric notions is the chain problem that asks if any maximal chain of prime ideals in a Noetherian local integral domain R has the same length. Nagata introduced two chain conditions. The first chain condition is that the length of any maximal chain of R is equal to $\dim R$, and the second chain condition is that the first chain condition hold in any integral extension of R. The two conditions become equivalent if the ring under consideration is normal. These conditions lead to the notion of catenary ring or universally catenary rings (cf. [R7]) and to a notion of pseudo-geometric rings (see [24], [41]). The subject is also connected to the study of the unmixedness theorem and of Cohen-Macaulay rings. In [24], Nagata constructed examples for which the first or the second chain condition does not hold. Ogoma [R8] later constructed examples of pseudo-geometric normal local rings which do not satisfy the second chain condition. In [41], he raised a question that is linked to a problem asking whether the zero ideal of the completion of a Noetherian local integral domain has an embedded prime ideal. This is one of the questions posed by Nagata which motivated later developments in the research on commutative rings. Counterexamples to this question were given first by Ferrand and Raynaud [R2] and later by Nagata [B13].

Modeling geometric local rings, Nagata [12] introduced the notion of pseudo-geometric rings, which are called *Nagata rings* in [R7] and *universally Japanese rings* in [R3]. When Nagata considered these rings, he was probably influenced by the work of Zariski and Chevalley and thinking of algebraic variety defined over a ground ring. Pseudo-geometric local integral domains behave nicely like geometric local rings, and they are analytically unramified. Furthermore, Nagata viewed an algebraic variety as a collection of localities. A *locality* is defined to be an integral domain which is a localization of a finitely generated algebra over a ground ring. With this notion of localities, Nagata [26], [36], [42] developed a general theory of algebraic geometry over Dedekind domains. One can view these works of Nagata as precursors to schemes introduced by Grothendieck [R3].

In 1962, Nagata published the famous textbook on local rings [B3] which comprised all basic results as well as the most advanced results and which, together with the homological methods introduced by Serre [R12] and Grothendieck [R5], laid the foundation for the study of local rings.

2. General theory of algebraic geometry and related ring theory

In [26], [36], and [42], Nagata developed a theory of algebraic varieties over a field or a Dedekind domain I with an occasional assumption that the integral closure of I in any finite algebraic extension of the quotient field $Q(I)$ is a finite I-module. The theory was modeled after the one over a ground field given by Chevalley during his lecture at Kyoto University in 1954. Nagata defined a spot (synonymous with a locality) to be a local ring $R_{\mathfrak{p}}$, where R is an affine ring over I and $\mathfrak{p}$ is a prime ideal. Given an affine ring R over I, an affine model with R as the coordinate ring is the collection of local rings $R_{\mathfrak{p}}$, where $\mathfrak{p}$ runs through all prime ideals of R. Hence an affine model is essentially an affine scheme $\operatorname{Spec} R$. Other models including complete and projective models were defined through patching affine models. Nagata wrote a book, jointly with Y. Nakai [B1], from this viewpoint on algebraic geometry in the case where the ground ring is a field.

In [25], Nagata developed the theory of multiplicities after Chevalley and Samuel by making use of Hilbert characteristic function. In [18], the intersection multiplicity of two subvarieties properly meeting in an algebraic variety was formulated in terms of local rings. In [31], when $A = k[[x_1, \ldots, x_n]]$ is a formal power series ring over a field k of characteristic $p \neq 0$ and $R = A_{\mathfrak{q}}$ for a prime ideal $\mathfrak{q}$, a criterion for regularity of $R/\mathfrak{p}R$ for a prime ideal $\mathfrak{p}$ was given in terms of the mixed Jacobian matrix of a set of generators of $\mathfrak{p}$. In [44], the purity of branch loci was proved for a general regular local ring R and its normal overring Q which is a finite integral extension of R, while the geometric purity theorem was given by Zariski and the ring-theoretic version was proved by Auslander and Buchsbaum in dimension two. In [45], Nagata considered the closedness of the singular loci of affine models over the ground Dedekind domain satisfying the "finiteness condition." In [22], it was proved that if V is a normal projective variety, then the Chow variety $V^{(m)}$ of zero-cycles of degree m is normal as well.

3. The fourteenth problem of Hilbert

The fourteenth problem of Hilbert in its original form asked whether $k[x_1, \ldots, x_n] \cap L$ is finitely generated over k, where $k[x_1, \ldots, x_n]$ is a polynomial ring over a field k and L is a subfield of $k(x_1, \ldots, x_n)$. Nagata solved this problem negatively by producing a counterexample and was invited to give a talk at the International Congress of Mathematicians held at Edinburgh in 1958.

It is not clear when Nagata started thinking about the problem. It was probably after he moved to Kyoto University from Nagoya in 1953. The first publication by Nagata on the problem was his dissertation [27]. In this work, he introduced the notion of ideal transforms for a normal ring and its ideal, generalizing Zariski's treatment in terms of a variety and a divisor (see [R15]). Nagata asked the following question: *Let R be a normal affine ring over a ground ring, and let L be a subfield of the quotient field of R. Is $R \cap L$ finitely generated over I?* Although this is a slight generalization of Zariski's original formulation of the fourteenth problem (when I is a field), Nagata proved that the thus-formulated problem is equivalent to the finite generation of ideal transforms. It was Rees [R9] who produced a counterexample to Zariski's formulation just by making use of Nagata's notion of ideal transforms. Rees gave a ring of dimension three making use of a smooth cubic plane curve over the complex field $\mathbb{C}$.

In 1958, Nagata [48] gave a counterexample to the original problem, which we now explain with slight modifications as follows. Let $\mathbb{C}^{\oplus n}$ be a direct product of n-copies of the additive group of the complex field $\mathbb{C}$. Consider a linear action of $\mathbb{C}^{\oplus n}$ on a $2n$-dimensional polynomial ring $R = \mathbb{C}[x_1, \ldots, x_n, y_1, \ldots, y_n]$ given by $x_i \mapsto x_i, y_i \mapsto y_i + t_i x_i$ for $1 \leq i \leq n$ and $t = (t_1, \ldots, t_n) \in \mathbb{C}^{\oplus n}$. Then for a subgroup G of $\mathbb{C}^{\oplus n}$ defined by three generic linear equations and $n = 16$, the invariant subring R^G of dimension 19 is not finitely generated over $\mathbb{C}$. Nagata obtained this example by observing the geometry of plane curves (see also [46]). After this counterexample given by Nagata, efforts were made to find counterexamples in lower dimension or those obtained as the invariant subrings of the additive group G_a acting nonlinearly on a polynomial ring. Roberts [R10] gave a counterexample of dimension 6 based on a nonlinear G_a-action (expressed in terms of a locally nilpotent derivation) which was very influential. Kuroda [R6] gave a counterexample with $\dim L = 3$. The lecture notes [B4] are a good source of information on this subject including the basic results in invariant theory.

As for the finite generation of such rings $R \cap L$, Nagata considered some sufficient conditions in [59], [62], and [63]. It is interesting to note that he gave a sufficient condition in [62] in terms of strong submersiveness of $R \cap L$ in R. This idea led him to his works in invariant theory discussed in Section 6.

4. Complete embedding theorem

First, Nagata was interested in asking whether any algebraic variety can be embedded biregularly into a projective variety, and he constructed in [28] an example for which such an embedding is impossible, and then he constructed in [39] the renowned examples of

(1) a nonprojective complete normal algebraic variety of dimension larger than or equal to two,

(2) a nonprojective complete nonsingular algebraic variety of dimension larger than or equal to three defined over the prime field.

Then he proved that any algebraic variety can be embedded into a complete variety as an open set (see [53]) and that any Noetherian scheme of finite type over a Noetherian ground scheme can be embedded as an open set into a proper scheme over the same ground scheme (see [55]). In the second paper, he used the language of schemes for the first time. Sumihiro [R14] remade the theorem in the equivariant setting.

5. Rational surfaces

In [49] and [50], Nagata gave

(1) the classification of relatively minimal nonsingular rational surfaces (Hirzebruch surfaces F_n and the projective plane, in other words),

(2) the classification of rational ruled surfaces, and

(3) factorization of Cremona plane transformations.

He showed that any (not necessarily rational) ruled surface is obtained from $C \times \mathbb{P}^1$ by successively applying elementary transformations. Furthermore, he described in detail the images (strict transforms) of plane curves with preassigned base points and multiplicities under the blowups of these base points and counted the number of exceptional curves of the first kind on the surface obtained from $\mathbb{P}^2$ with base points blown up. In [70], he showed that any relatively minimal ruled surface over a curve of genus $g \geq 0$ has a minimal section with self-intersection number $\leq g$.

6. Invariant theory

Extending the complete reducibility problem of linear representations of a reductive algebraic group to the case of characteristic $p > 0$, Nagata first proved in [52] that in the case of positive characteristic p, every rational representation of an algebraic group G is completely reducible; that is, G is linearly reductive if and only if the connected component G^0 is an algebraic torus and the index $[G : G^0]$ is prime to p. In [54], he considered the construction of quotient variety in the sense that two points P, Q of a G-variety are equivalent if and only if the closures of the G-orbits of P, Q intersect. For a G-variety and a point $P \in V$, identify P with the local ring $\mathcal{O}_{V,P}$, and denote $P_G = P \cap k(V)^G$. He discussed the conditions which guarantee that P_G is a locality, that is, a local ring of a point of a variety. When the characteristic is zero, it is shown that in the case of a semisimple group action on a projective variety V, the quotient variety exists as a quasi-projective variety under some additional conditions on the projective embedding and the G-action.

In [56], he defined a semireductive (now said to be *geometrically reductive*) group and drew attention to the so-called Mumford conjecture, which was later solved by Haboush [R4]. It was shown that if a semireductive group G acts on an affine domain R over a field, then R^G is finitely generated. In [57], Nagata and T. Miyata proved that a semireductive group is reductive, while the Mumford conjecture (Haboush's theorem) asserts that the converse is true. In [58] and [61], the above result on finite generation for a semireductive group action on an affine domain over a field was extended to the case where the ground field k is replaced by a pseudo-geometric ring K with a minor restriction on K.

7. Polynomial rings and related results

In [47], Nagata gave a negative answer to a problem of Abhyankar: *For a local ring R over an imperfect field k and a purely inseparable extension k' of k of degree p, if the normalization R' of $R \otimes_k k'$ is a regular local ring, is R then regular?* Nagata's counterexample was the local ring of a point on a hypersurface which is a degree p extension of the affine plane $\mathbb{A}^2$. In [68], it was proved that if R is a ring and $A = R[X]/(f(X))$ is the coordinate ring of a hypersurface $f(X) = a_0X^{(0)} + a_1X^{(1)} + \cdots + a_nX^{(n)} = 0$ with $a_i \in R$, where $X = (X_1, \ldots, X_r)$ is a set of variables and the $X^{(i)}$ are the monomials with $X^{(i)} \neq X^{(j)}$ $(i \neq j)$, then A is R-flat if and only if the ideal J generated by the coefficients $a_0, \ldots, a_n$ is a principal ideal $J = eR$ with $e^2 = e$. Nagata wrote several short articles concerning ring extensions and field extensions, for example, [35], [64], and [82]. In [94] and [95], he proved a result called Eakin-Nagata theorem and its generalization, that if A is a subring of a Noetherian ring R and if R is a finite A-module, then A is Noetherian.

In [71], he gave a new proof to a theorem of Gutwirth, which is stated as follows. Let C be a projective plane curve of degree d such that $C \cap \mathbb{A}^2$ is isomorphic to the affine line $\mathbb{A}^1$, where $\mathbb{A}^2 = \mathbb{P}^2 \setminus D$ with a line D. Let $P_1, \ldots, P_n$

be the singular points (of multiplicity m_i, respectively) of C lying on D, where P_1 is an ordinary point and P_{i+1} is infinitely near to P_i. If the ground field has characteristic zero and the linear system consisting of plane curves of degree d with base conditions $\sum_{i=1}^{n} m_i P_i$ has positive dimension, then d is a multiple of $d - m_1$, and $k[x,y] = k[f,g]$, where $\{x,y\}$ is a system of coordinates on $\mathbb{A}^2$ and g is a polynomial in x, y. Indeed, this result holds without the condition on the linear system, and it is called the Abhyankar-Moh-Suzuki theorem in affine algebraic geometry (see [R1]).

In 1972, Nagata wrote a monograph [B12] entitled *On automorphism group of* $k[x,y]$ and discussed the structure of the automorphism group of a polynomial ring defined over a ring k. When k is a field, the structure is known as a Jung–van der Kulk theorem which asserts that $\mathrm{Aut}_k\, k[x,y]$ is an amalgamated product of the subgroup of linear transformations and the subgroup of de Jonquière transformations. Nagata conjectured that in dimension three over a field k, the automorphism σ defined by $\sigma(x,y,z) = (x - 2y(zx+y^2) - z(zx+y^2)^2, y + z(zx+y^2), z)$ is not in the subgroup generated by linear transformations and de Jonquière transformations; that is, σ is a wild automorphism. The conjecture was recently solved by Shestakov and Umirbaev [R13]. It took almost thirty years, but Nagata's insight shed light on the study of polynomial automorphisms.

Nagata also considered the Jacobian conjecture in dimension two (see [91], [92]).

References

[R1] S. S. Abhyankar and T. T. Moh, *Embeddings of the line in the plane*, J. Reine Angew. Math. **276** (1975), 148–166.

[R2] D. Ferrand and M. Raynaud, *Fibres formelles d'un anneau local noethérien*, Ann. Sci. École Norm. Sup. (4) **3** (1970), 295–311.

[R3] A. Grothendieck and J. Dieudonné, Éléments de géométrie algébrique, Publ. Math. IHES. **4, 8, 11, 17, 20, 24, 28, 32**.

[R4] W. J. Haboush, *Reductive groups are geometrically reductive*, Ann. of Math. (2) **102** (1975), 67–83.

[R5] R. Hartshorne, *Local Cohomology: A Seminar Given by A. Grothendieck, Harvard University, Fall 1961*, Lecture Notes in Mathematics **41**, Springer, Berlin, 1967.

[R6] S. Kuroda, *A counterexample to the fourteenth problem of Hilbert in dimension three*, Michigan Math. J. **53** (2005), 123–132.

[R7] H. Matsumura, *Commutative Ring Theory*, Cambridge Stud. Adv. Math. **8**, Cambridge Univ. Press, Cambridge, 1986.

[R8] T. Ogoma, *Noncatenary pseudogeometric normal rings*, Japan. J. Math. (N.S.) **6** (1980), 147–163.

[R9] D. Rees, *On a problem of Zariski*, Illinois J. Math. **2** (1958), 145–149.

[R10] P. Roberts, *An infinitely generated symbolic blow-up in a power series ring and a new counterexample to Hilbert's fourteenth problem*, J. Algebra **132** (1990), 461–473.

[R11] P. Samuel, *Lectures on unique factorization domains* (notes by Pavman Murthy), Tata Institute for Fundamental Research lectures, **30**, Bombay, Tata Institute, 1964.

[R12] J.-P. Serre, *Algèbre locale: Multiplicités*, 2nd ed., Lecture Notes in Math. **11**, Springer, Berlin, 1965.

[R13] I. P. Shestakov and U. U. Umirbaev, *The tame and the wild automorphisms of polynomial rings in three variables*, J. Amer. Math. Soc. **17** (2004), 197–227.

[R14] H. Sumihiro, *Equivariant completion, I*, J. Math. Kyoto Univ. **14** (1974), 1–28; *II*, J. Math. Kyoto Univ. **15** (1975), 573–605.

[R15] O. Zariski, *Interprétations algébrico-géométriques du quatorzième problème de Hilbert*, Bull. Sci. Math. (2) **78** (1954), 155–168.

Other cited works and bibliography

The following is the list of research articles, books and monographs, and expositions and short notes published in the journal *Sûgaku*. The author owes thanks to Jun-Ichi Nishimura for the file of the complete works of Nagata.

Research articles

[1] N. Ito and M. Nagata, *Note on groups of automorphisms*, Kodai Math. Sem. Rep. **1** (1949), 37–39.

[2] M. Nagata, *On the structure of complete local rings*, Nagoya Math. J. **1** (1950), 63–70.

[3] ———, *On the theory of semi-local rings*, Proc. Japan Acad. **26** (1950), nos. 2-5, 131–140.

[4] ———, *Some studies on semi-local rings*, Nagoya Math. J. **3** (1951), 23–30.

[5] ———, *On the theory of radicals in a ring*, J. Math. Soc. Japan **3** (1951), 330–344.

[6] ———, *Note on subdirect sums of rings*, Nagoya Math. J. **2** (1951), 49–53.

[7] ———, *On Krull's conjecture concerning valuation rings*, Nagoya Math. J. **4** (1952), 29–33.

[8] ———, *On the nilpotency of nil-algebras*, J. Math. Soc. Japan **4** (1952), 296–301.

[9] ———, *Note on groups with involutions*, Proc. Japan Acad. **28** (1952), 564–566.

[10] ———, *Corrections to my paper* "On the structure of complete local rings," Nagoya Math. J. **5** (1953), 145–147.

[11] ———, *On the theory of Henselian rings*, Nagoya Math. J. **5** (1953), 45–57.

[12] ———, *Some remarks on local rings*, Nagoya Math. J. **6** (1953), 53–58.

[13] ______, T. Nakayama, and T. Tuzuku, *On an existence lemma in valuation theory*, Nagoya Math. J. **6** (1953), 59–61.

[14] ______, *Some remarks on local rings, II*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **28** (1954), 109–120.

[15] ______, *Note on complete local integrity domains*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **28** (1954), 271–278.

[16] ______, *On the theory of Henselian rings, II*, Nagoya Math. J. **7** (1954), 1–19.

[17] ______, *Note on integral closures of Noetherian domains*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **28** (1954), 121–124.

[18] ______, *Note on intersection multiplicity of proper components of algebraic or algebroid varieties*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **28** (1954), 279–281.

[19] ______, *Basic theorems on general commutative rings*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **29** (1955), 59–77.

[20] ______, *An example of normal local ring which is analytically ramified*, Nagoya Math. J. **9** (1955), 111–113.

[21] ______, *On the derived normal rings of Noetherian integral domains*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **29** (1955), 293–303.

[22] ______, *On the normality of the Chow variety of positive 0-cycles of degree m in an algebraic variety*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **29** (1955), 165–176.

[23] ______, *Corrections to my paper* "On Krull's conjecture concerning valuation rings," Nagoya Math. J. **9** (1955), 209–212.

[24] ______, *On the chain problem of prime ideals*, Nagoya Math. J. **10** (1956), 51–64.

[25] ______, "The theory of multiplicity in general local rings" in *Proceedings of the International Symposium on Algebraic Number Theory (Tokyo & Nikko, 1955)*, Science Council of Japan, Tokyo, 1956, 191–226.

[26] ______, *A general theory of algebraic geometry over Dedekind domains, I: The notion of models*, Amer. J. Math. **78** (1956), 78–116.

[27] ______, *A treatise on the 14-th problem of Hilbert*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1956), 57–70.

[28] ______, *On the imbedding problem of abstract varieties in projective varieties*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1956), 71–82.

[29] ______, *A remark on the unique factorization theorem*, J. Math. Soc. Japan **9** (1957), 143–145.

[30] ______, *Note on a paper of Samuel concerning asymptotic properties of ideals*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1957), 165–175.

[31] ______, *A Jacobian criterion of simple points*, Illinois J. Math. **1** (1957), 427–432.

[32] H. Matsumura and M. Nagata, *On the algebraic theory of sheets of an algebraic variety*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1957), 157–164.

[33] M. Nagata, *Addition and corrections to my paper* "A treatise on the 14-th problem of Hilbert," Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1957), 197–200.

[34] ———, *On the imbeddings of abstract surfaces in projective varieties*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1957), 231–235.

[35] ———, *Note on a paper of Lang concerning quasi algebraic closure*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **30** (1957), 237–241.

[36] ———, *A general theory of algebraic geometry over Dedekind domains, II: Separably generated extensions and regular local rings*, Amer. J. Math. **80** (1958), 382–420.

[37] ———, *An example of a normal local ring which is analytically reducible*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **31** (1958), 83–85.

[38] ———, *Remarks on a paper of Zariski on the purity of branch loci*, Proc. Nat. Acad. Sci. U.S.A. **44** (1958), 796–799.

[39] ———, *Existence theorems for nonprojective complete algebraic varieties*, Illinois J. Math. **2** (1958), 490–498.

[40] ———, *On the theory of Henselian rings, III*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **32** (1959), 93–101.

[41] ———, *Note on a chain condition for prime ideals*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **32** (1959), 85–90.

[42] ———, *A general theory of algebraic geometry over Dedekind domains, III: Absolutely irreducible models, simple spots*, Amer. J. Math. **81** (1959), 401–435.

[43] ———, *Note on coefficient fields of complete local rings*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **32** (1959), 91–92.

[44] ———, *On the purity of branch loci in regular local rings*, Illinois J. Math. **3** (1959), 328–333.

[45] ———, *On the closedness of singular loci*, Inst. Hautes Études Sci. Publ. Math. **2** (1959), 29–36.

[46] ———, *On the 14-th problem of Hilbert*, Amer. J. Math. **81** (1959), 766–772.

[47] ———, *An example to a problem of Abhyankar*, Amer. J. Math. **81** (1959), 501–502.

[48] ———, "On the fourteenth problem of Hilbert" in *Proceedings of the International Congress of Mathematicians, 1958*, Cambridge Univ. Press, New York, 1960, 459–462.

[49] ———, *On rational surfaces, I: Irreducible curves of arithmetic genus* 0 *or* 1, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **32** (1960), 351–370.

[50] ———, *On rational surfaces, II*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **33** (1960/1961), 271–293.

[51] ______, *Some remarks on prime divisors*, Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **33** (1960/1961), 297–299.

[52] ______, *Complete reducibility of rational representations of a matric group*, J. Math. Kyoto Univ. **1** (1961/1962), 87–99.

[53] ______, *Imbedding of an abstract variety in a complete variety*, J. Math. Kyoto Univ. **2** (1962), 1–10.

[54] ______, *Note on orbit spaces*, Osaka Math. J. **14** (1962), 21–31.

[55] ______, *A generalization of the imbedding problem of an abstract variety in a complete variety*, J. Math. Kyoto Univ. **3** (1963), 89–102.

[56] ______, *Invariants of a group in an affine ring*, J. Math. Kyoto Univ. **3** (1963/1964), 369–377.

[57] ______ and T. Miyata, *Note on semi-reductive groups*, J. Math. Kyoto Univ. **3** (1963/1964), 379–382.

[58] ______ and T. Miyata, *Remarks on matric groups*, J. Math. Kyoto Univ. **4** (1965), 381–384.

[59] ______ and K. Otsuka, *Some remarks on the 14th problem of Hilbert*, J. Math. Kyoto Univ. **5** (1965), 61–66.

[60] ______, *Finitely generated rings over a valuation ring*, J. Math. Kyoto Univ. **5** (1966), 163–169.

[61] ______, *Invariants of a group under a semi-reductive action*, J. Math. Kyoto Univ. **5** (1966), 171–176.

[62] ______, *A theorem on finite generation of a ring*, Nagoya Math. J. **27** (1966), 193–205.

[63] ______, "Some sufficient conditions for the fourteenth problem of Hilbert" in *Proceedings of the International Colloquium on Algebraic Geometry (Madrid, 1965)* (in Spanish), Inst. Jorge Juan del C.S.I.C.-Internat. Math. Union, Madrid, 1966, 107–121.

[64] ______, *A theorem on valuation rings and its applications*, Nagoya Math. J. **29** (1967), 85–91.

[65] ______, *A type of integral extensions*, J. Math. Soc. Japan **20** (1968), 266–267.

[66] ______, *A type of subrings of a noetherian ring*, J. Math. Kyoto Univ. **8** (1968), 465–467.

[67] ______, "Some questions on rational actions of groups" in *Algebraic Geometry (Bombay, 1968)*, Oxford Univ. Press, London, 1969, 323–334.

[68] ______, *Flatness of an extension of a commutative ring*, J. Math. Kyoto Univ. **9** (1969), 439–448.

[69] ______ and M. Maruyama, *Note on the structure of a ruled surface*, J. Reine Angew. Math. **239/240** (1969), 68–73.

[70] ______, *On self-intersection number of a section on a ruled surface*, Nagoya Math. J. **37** (1970), 191–196.

[71] ———, *A theorem of Gutwirth*, J. Math. Kyoto Univ. **11** (1971), 149–154.

[72] M. Artin and M. Nagata, *Residual intersections in Cohen-Macaulay rings*, J. Math. Kyoto Univ. **12** (1972), 307–323.

[73] M. Nagata, *Some remarks on ordered fields*, Japan. J. Math. (N.S.) **1** (1975/76), 1–4.

[74] ———, "Some remarks on ordered fields" in *Séminaire P. Dubreil, F. Aribaud et M.-P. Malliavin (28e année: 1974/75): Algèbre*, Secrétariat Matématique, Paris, 1975, exp. no. 26.

[75] ———, *Some questions on Cohen-Macaulay rings*, J. Math. Kyoto Univ. **13** (1973), 123–128.

[76] ———, *Some types of simple ring extensions*, Houston J. Math. **1** (1975), 131–136.

[77] ———, *Subrings of a polynomial ring of one variable*, J. Math. Kyoto Univ. **17** (1977), 511–512.

[78] T. Akiba and M. Nagata, *On normality of a Noetherian ring*, J. Math. Kyoto Univ. **17** (1977), 605–609.

[79] M. Nagata, "On Euclid algorithm" in *C. P. Ramanujam—a Tribute*, Tata Inst. Fund. Res. Studies in Math. **8**, Springer, Berlin, 1978, 175–186.

[80] ———, *A generalization of the notion of a valuation*, Amer. J. Math. **101** (1979), 245–257.

[81] ———, *Commutativity of elements in an amalgamated product*, Japan. J. Math. (N.S.) **6** (1980), 173–178.

[82] ———, *A field extension with certain finiteness condition on multiplicative group extension*, J. Math. Kyoto Univ. **22** (1982/83), 255–256.

[83] ———, "Applications of the theory of valuation rings" in *Proceedings of the International Mathematical Conference, Singapore 1981 (Singapore, 1981)*, North-Holland Math. Stud. **74**, North-Holland, Amsterdam, 1982, 3–8.

[84] ———, "Commutative algebra and algebraic geometry" in *Proceedings of the International Mathematical Conference, Singapore 1981 (Singapore, 1981)*, North-Holland Math. Stud. **74**, North-Holland, Amsterdam, 1982, 125–154.

[85] ———, *Some problems on linear systems of plane curves*, Chinese J. Math. **11** (1983), 1–4.

[86] ———, *A conjecture of O'Carroll and Qureshi on tensor products of fields*, Japan. J. Math. (N.S.) **10** (1984), 375–377.

[87] ———, *Some remarks on Euclid rings*, J. Math. Kyoto Univ. **25** (1985), 421–422.

[88] ———, "On the definition of a Euclid ring" in *Commutative Algebra and Combinatorics (Kyoto, 1985)*, Adv. Stud. Pure Math. **11**, North-Holland, Amsterdam, 1987, 167–171.

[89] A. Nowicki and M. Nagata, *Rings of constants for k-derivations in $k[x_1, \dots, x_n]$*, J. Math. Kyoto Univ. **28** (1988), 111–118.

[90] M. Nagata, “A pairwise algorithm and its application to $Z[\sqrt{14}]$” in *Algebraic Geometry Seminar (Singapore, 1987)*, World Sci., Singapore, 1988, 69–74.

[91] ______, “Two-dimensional Jacobian conjecture” in *Algebra and Topology (Taejon, Korea, 1988)*, Korea Inst. Tech., Taejon, 1988, 77–98.

[92] ______, *Some remarks on the two-dimensional Jacobian conjecture*, Chinese J. Math. **17** (1989), 1–7.

[93] ______, “Pairwise algorithms and Euclid algorithms” in *Collection of Papers Dedicated to Prof. Jong Geun Park on His Sixtieth Birthday*, Jeonbug, Seoul, Korea, 1989, 1–9.

[94] ______, *A proof of the theorem of Eakin-Nagata*, Proc. Japan Acad. Ser. A Math. Sci. **67** (1991), 238–239.

[95] ______, *A new proof of the theorem of Eakin-Nagata*, Chinese J. Math. **20** (1992), 1–3.

[96] ______, *On Eakin-Nagata-Formanek theorem*, J. Math. Kyoto Univ. **33** (1993), 825–826.

[97] ______, “Some questions on $Z[\sqrt{14}]$” in *Algebraic Geometry and its Applications (West Lafayette, Ind., 1990)*, Springer, New York, 1994, 327–332.

[98] ______, *On Zariski's problem concerning the 14th problem of Hilbert*, Osaka J. Math. **33** (1996), 997–1002.

Books and monographs

[B1] Y. Nakai and M. Nagata, *Algebraic Geometry* (in Japanese), Lectures in Contemp. Math. **16**, Kyōritsu Shuppan, Tokyo, 1957.

[B2] Y. Akizuki and M. Nagata, *Modern Algebra* (in Japanese), Ser. of Modern Math., Kyoritsu-Shuppan, Tokyo, 1957.

[B3] M. Nagata, *Local Rings*, Intersci. Tracts Pure Appl. Math. **13**, Intersci., New York, 1962.

[B4] ______, *Lectures on the Fourteenth Problem of Hilbert*, Tata Inst. Fund. Research, Bombay, 1965.

[B5] A. Komatsu and M. Nagata, *Algebra and Geometry for Science and Technology* (in Japanese), Kyoritsu Shuppan, Tokyo, 1966.

[B6] M. Nagata, *Theory of Commutative Fields* (in Japanese), Shokabo, Tokyo, 1967.

[B7] ______, *Introduction to Abstract Algebra* (in Japanese), Asakura Shoten, Tokyo, 1967.

[B8] ______, *Algebraic Groups* (in Japanese), Kyoritsu Shuppan, Tokyo, 1969.

[B9] ______, *Introduction to Set Theory* (in Japanese), Morikita Shuppan, Tokyo, 1970.

[B10] ______, *On Flat Extensions of a Ring*, Sém. Math. Supérieures **46** (été 1970), Presses de l'Univ. Montréal, Montreal, 1971.

[B11] ______, M. Miyanishi, and M. Maruyama, *Abstract Algebraic Geometry* (in Japanese), Kyoritsu Shuppan, Tokyo, 1972.

[B12] ______, *On Automorphism Group of* $k[x, y]$, Kyoto Univ. Lectures in Math. **5**, Kinokuniya Book-Store, Tokyo, 1972.

[B13] ______, *Theory of Commutative Rings* (in Japanese), Kinokuniya Book-Store, Tokyo, 1974.

[B14] ______, *Field Theory*, Pure and Appl. Math. **40**, Dekker, New York, 1977.

[B15] ______, *Polynomial Rings and Affine Spaces*, Regional Conference Ser. Math. **37**, Amer. Math. Soc., Providence, 1978.

[B16] ______, *Basic Linear Algebra for Science Students* (in Japanese), Kinokuniya, Tokyo, 1987.

[B17] ______, *Theory of Commutative Fields*, trans. from the 1985 Japanese edition by the author, Trans. Math. Monogr. **125**, Amer. Math. Soc., Providence, 1993.

[B18] ______ and K.-I. Yoshida, *Introduction to Algebra*, Baifuukan, Tokyo, 1996.

[B19] ______, *Algebraic Geometry for High School Students* (in Japanese), Gendai-Sugaku-Sha, Kyoto, 1997.

[B20] ______, *Invitation to Group Theory* (in Japanese), Gendai-Sugaku-Sha, Kyoto, 2007.

Expositions and short notes published in the journal *Sûgaku*

[S1] M. Nagata, *An ideal-theoretic observation on valuations* (in Japanese), Sûgaku **4** (1952), 76–80.

[S2] ______, *Local rings, I* (in Japanese), Sûgaku **5** (1953), 104–114.

[S3] ______, *Local rings, II* (in Japanese), Sûgaku **5** (1953), 229–238.

[S4] ______, *On the 14th problem of Hilbert* (in Japanese), Sûgaku **12** (1960/1961), 203–209.

[S5] ______ and H. Matsumura, *A theorem in elementary arithmetic* (in Japanese), Sûgaku **13** (1961/1962), 161.

[S6] ______, *On* $\mathrm{SL}(n, K)$ (in Japanese), Sûgaku **13** (1961/1962), 108.

[S7] ______, *On the number of solutions of* $x_1^2 + x_2^2 + \cdots + x_n^2 = a$ *in a finite field* (in Japanese), Sûgaku **14** (1962/1963), 98–99.

[S8] ______, *A warning on zero divisors* (in Japanese), Sûgaku **21** (1969), 131.

[S9] ______, *On the rank of a maximal free subgroup*, Sûgaku **21** (1969), 130–131.

[S10] ______, *A problem on the existence of prime ideals* (in Japanese), Sûgaku **27** (1975), 368.

[S11] ______, 50 *years of commutative ring theory* (in Japanese), Sûgaku **36** (1984), 157–163.

[S12] ______, *A generalization of the Fibonacci sequence* (in Japanese), Sûgaku **46** (1994), 69–71.

[S13] ———, *A generalization of the Fibonacci sequence, II* (in Japanese), Sûgaku **46** (1994), 358–360.

[S14] ———, *Certain problems on the difference between n individual numbers from 2 groups* (in Japanese), Sûgaku **49** (1997), 214–217.

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