COMPARISON OF EXPERIMENTS

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1. Summary

Bohnenblust, Shapley, and Sherman [2] have introduced a method of comparing two sampling procedures or experiments; essentially their concept is that one experiment α is more informative than a second experiment β, $\alpha \supset \beta$, if, for every possible risk function, any risk attainable with β is also attainable with α. If α is a sufficient statistic for a procedure equivalent to β, $\alpha \succ \beta$, it is shown that $\alpha \supset \beta$. In the case of dichotomies, the converse is proved. Whether $\succ$ and $\supset$ are equivalent in general is not known. Various properties of $\succ$ and $\supset$ are obtained, such as the following: if $\alpha \succ \beta$ and γ is independent of both, then the combination $(\alpha, \gamma) \succ (\beta, \gamma)$. An application to a problem in 2×2 tables is discussed.

2. Definitions

An *experiment* α is a set of N probability measures $u_1, \ldots, u_N$ on a Borel field B of subsets of a space X. The N measures are considered as N possible distributions over X, and performing the experiment consists of observing a sample point $x \in X$. A *decision problem* is a pair (α, A), where A is a bounded subset of N-space. The points $a \in A$ are considered as the possible actions open to the statistician; the loss from action $a = (a_1, \ldots, a_N)$ is a_i if the actual distribution of x is u_i. A *decision procedure* f for (α, A) is a B-measurable function from X into A, specifying the action a to be taken as a function of the sample point x obtained by the experiment. With every $f = [a_1(x), \ldots, a_N(x)]$ is associated a loss vector

$$v(f) = \left(\int a_1(x)\, du_1, \ldots, \int a_N(x)\, du_N\right);$$

the i-th component of $v(f)$ is the expected loss from f if x has distribution u_i. The range of $v(f)$ is a subset of N-space which we denote by $R_1(\alpha, A)$; the convex closure of $R_1(\alpha, A)$ will be denoted by $R(\alpha, A)$ and will be called the set of *attainable loss vectors* in (α, A); every vector in R is either attainable or approximable by a randomized mixture of $N + 1$ decision procedures.

THEOREM 1. $R(\alpha, A) = R(\alpha, A_1) = R_1(\alpha, A_1)$, *where* A_1 *is the convex closure* of A.

This theorem permits us to restrict attention to closed convex A, which we shall do in the following sections. The proof of the theorem will not be given here; it is straightforward except for the fact that $R(\alpha, A_1) = R_1(\alpha, A_1)$. This fact follows from the result that whenever A is closed, so is $R_1(\alpha, A)$, which has been proved elsewhere by the author [1].

Following Bohnenblust, Shapley and Sherman [2], we shall say that α is *more informative* than β, written $\alpha \supset \beta$, if for every A we have $R(\alpha, A) \supset R(\beta, A)$.