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Stochastic Burgers and KPZ Equations from Particle Systems*

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Abstract: We consider two strictly related models: a solid on solid interface growth model and the weakly asymmetric exclusion process, both on the one dimensional lattice. It has been proven that, in the diffusive scaling limit, the density field of the weakly asymmetric exclusion process evolves according to the Burgers equation [8, 13, 18] and the fluctuation field converges to a generalized Ornstein-Uhlenbeck process [8, 10]. We analyze instead the density fluctuations beyond the hydrodynamical scale and prove that their limiting distribution solves the (non linear) Burgers equation with a random noise on the density current. For the solid on solid model, we prove that the fluctuation field of the interface profile, if suitably rescaled, converges to the Kardar–Parisi–Zhang equation. This provides a microscopic justification of the so called *kinetic roughening*, i.e. the non Gaussian fluctuations in some non-equilibrium processes. Our main tool is the Cole-Hopf transformation and its microscopic version. We also develop a mathematical theory for the macroscopic equations.

1. Introduction

The hydrodynamic behavior of physical systems is usually described by (non linear) PDE's. This description is in most of the cases approximate and, to model various neglected effects, a random forcing term can be added to the macroscopic equation. One is particularly interested in scale invariant forces, of which the space–time white noise is a typical example. This choice however introduces small scale singularities and poses the problem of the existence of the stochastic dynamics even when the deterministic equation is known to have good smoothing properties. Most rigorous results are restricted to one space dimension and several substantial problems appear in higher dimensions, see e.g. [1, 16]. On the other side, the question whether non linear stochastic PDE's, at

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