ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 6, Number 1, Winter 1976

THE RADON-NIKODYM THEOREM FOR BANACH SPACE VALUED MEASURES

J. DIESTEL AND J. J. UHL, JR.

The present notes are an updated version of a brief history of the Radon-Nikodym theorem for Banach space valued measures written by the first author in January, 1973. Since that time much progress has been made in this subject, and these notes are aimed at conveying some of the flavor of this progress and hopefully interesting readers in some of the problems that remain.

Our style will be informal. A few proofs are included. Complete details for most of what appears herein will be found in the finished version of [0].

There appears to be roughly three aspects to the theory of differentiation of vector-valued measures: analytic, operator theoretic and geometric. While these aspects necessarily are intimately interrelated, we shall try to discuss the Radon-Nikodym theorem for these three viewpoints separately. Our presentation will be, to a large extent, along historical lines, though occasionally we stray from this path.

In writing these notes, we have benefitted from conversations with many mathematicians. They have shown us examples and counterexamples, and have been kind enough to send us preprints of their related work (oftentimes they even sent handwritten copies of their work!). A nonexhaustive list includes: J. Batt, W. J. Davis, B. Faires, T. Figiel, A. Gleit, W. B. Johnson, P. Kranz, E. Leonard, D. R. Lewis, J. Lindenstrauss, R. H. Lohman, H. Maynard, P. Morris, R. R. Phelps, H. P. Rosenthal, C. Stegall and K. Sundaresan. To each we extend our gratitude. We were especially fortunate to have a number of long conversations with Bob Huff which were extremely beneficial and had a definite effect upon this version of these notes.

I. **Analytic Aspects of the Radon-Nikodym Theorem.** The start of the theory of vector-valued Radon-Nikodym theorems coincides (not too surprisingly) with the introduction of the first vector-valued integration theory by S. Bochner [1]. In this first paper on integration of vector-valued functions, Bochner notes that if every X-valued function of bounded variation defined on $[0, 1]$ is differentiable almost everywhere then each X-valued absolutely continuous function on $[0, 1]$ can be recovered from its derivative via the "Bochner" integral. It

Received by the editors on April 25, 1974.