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On $(n-1)$-dimensional projective spaces contained in the Grassmann variety $\mathrm{Gr}(n, 1)$

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§0. Introduction

In this paper, we understand by a variety a projective variety which is defined over a fixed algebraically closed field k of characteristic p (which can be zero).

Our main purpose of the present paper is to classify the type of subvarieties of $\mathrm{Gr}(n, 1)$ which are biregular to projective spaces of dimension $n-1$. [1)]

As examples of such varieties we know followings.

$$X_{n,1}^{0}=\left\{\begin{pmatrix}1, & 0, & 0,\dots, & 0\\ 0, & x_0, & x_1,\dots, & x_{n-1}\end{pmatrix}\in \mathrm{Gr}(n, 1)\,|\,(x_0, x_1,\dots, x_{n-1})\in \mathbf{P}^{n-1}\right\}^{2)}.$$

$$X_{n,1}^{1}=\left\{\begin{pmatrix}x_0, & x_1,\dots, & x_{n-1}, & 0\\ 0, & x_0,\dots, & x_{n-2}, & x_{n-1}\end{pmatrix}\in \mathrm{Gr}(n, 1)\,|\,(x_0, x_1,\dots, x_{n-1})\in \mathbf{P}^{n-1}\right\}.$$

$$\check{X}_{3,1}^{0}=\phi_3(X_{3,1}^{0})$$

$$\check{X}_{3,1}^{1}=\phi_3(X_{3,1}^{1})$$

where ϕ_n: $\mathrm{Gr}(n, 1)\to \mathrm{Gr}(n, n-2)$ is the dual biregular morphism.

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1) In general $\mathrm{Gr}(n, d)$ denotes the Grassman variety which paramerizes d-dimensional linear subspace of n-dimensional projective space $\mathbf{P}^n$.

2) By $\begin{pmatrix}1, & 0, & 0,\dots, & 0\\ 0, & x_0, & x_1,\dots, & x_{n-1}\end{pmatrix}$ we denote the point of $\mathrm{Gr}(n, 1)$ which represent the line which passes two points $(1, 0, 0,\dots, 0)$ and $(0, x_0, x_1,\dots, x_{n-1})$ of $\mathbf{P}^n$.