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Equivariant completion II

By

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0. Introduction.

In this paper, we shall generalize the results obtained in [13]. Let S be a scheme and let G be a surjective smooth affine group scheme over S with connected fibres and let X be a normal noetherian S-scheme on which G acts regularly. We shall prove following three results;

a) For any line bundle L on X, there is a positive integer m such that $L^m\,(=L^{\otimes m})$ is G-linearizable (cf. Theorem 1. 6). Moreover, if S is noetherian and if X is normal and quasi-projective over S, then there is a coherent $\mathcal{O}_S$-Module E (cf. Theorem 2. 5) such that

(1) There is an immersion $\varphi: X \to P(E)$,

(2) There is a representation $\rho: G \to \mathrm{Aut}_S(P(E))$ and

(3) The following diagram is commutative.

$$\begin{array}{ccc} G \underset{S}{\times} X & \xrightarrow{\quad\sigma\quad} & X \\ \Big\downarrow{\scriptstyle \rho\times\varphi} & & \Big\downarrow \\ \mathrm{Aut}_S(P(E)) \underset{S}{\times} P(E) & \xrightarrow{\quad\sigma'\quad} & P(E) \end{array}$$

where $\sigma: G \underset{S}{\times} X \to X$ is the regular action of G on X and σ': $\mathrm{Aut}_S(P(E)) \times P(E) \to P(E)$ is the canonical action of $\mathrm{Aut}_S(P(E))$ on $P(E)$. Therefore, the regular action G on X is linear.

b) If S is normal, noetherian and if X is an S-scheme satisfying the property (N) (cf. Definition 3. 5) on which G acts regularly, then X is covered by G-stable open subschemes $(U_i)_{1\leq i\leq n}$ which are